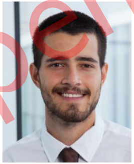
RESILIENT CONTROL: COMPROMISING TO ADAPT


L.F.O. Chamon


A. Amice


S. Paternain


A. Ribeiro

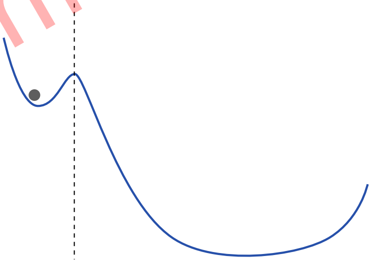
CDC 2020

December 14–18, 2020

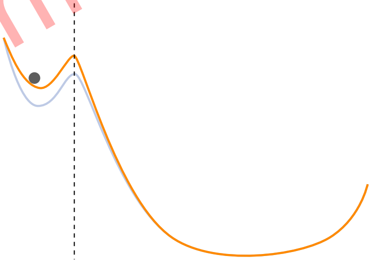
Dealing with uncertainty

- Sources of uncertainty:
  - Initial condition
  - Disturbances
  - Model mismatch

- Effects of uncertainty:
  - Deteriorate performance
  - Violate constraints



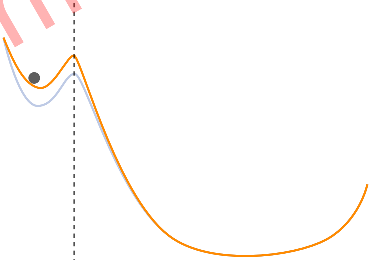
- Design the system to achieve its objective regardless of the operating conditions



Robustness

- Design the system to achieve its objective regardless of the operating conditions

Robust = hard to break

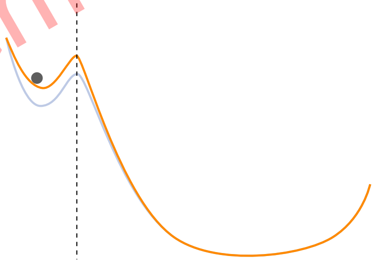


Robustness

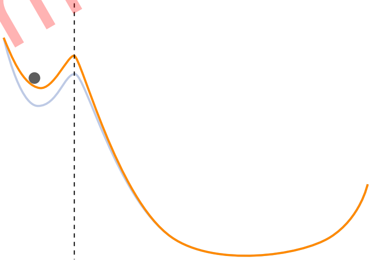
- Design the system to achieve its objective regardless of the operating conditions

Robust = hard to break

- **Methods:** $\mathcal{H}_\infty$ [DP'13], tube MPC [BBM'17], robust system-level synthesis [ADLM, ARC'19]



✓ Guaranteed to operates under specifications



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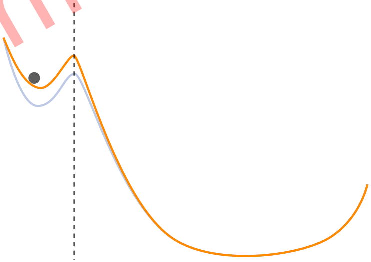
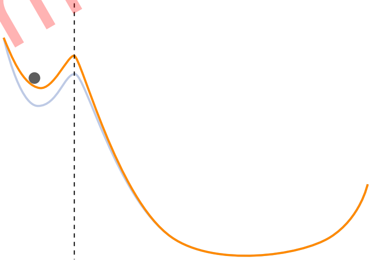
✓ Guaranteed to operates under specifications

✗ Poor nominal performance

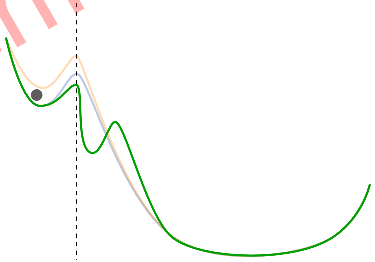


- ✓ Guaranteed to operates under specifications
- × Poor nominal performance
- × Infeasibility



Resilience

- Ecology: ability to adapt and recover from disruptions by modifying underlying behavior



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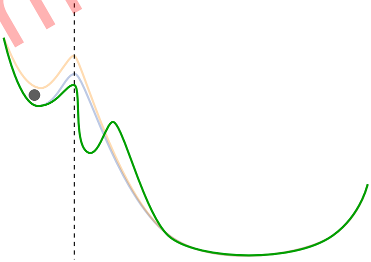
Resilience = easy to fix



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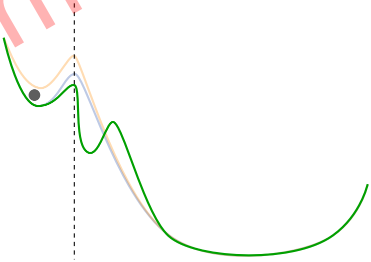
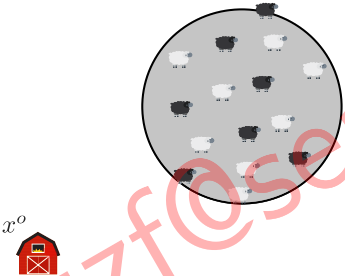
Resilience = easy to fix

- **Methods:** *ad hoc* [RPS IROS'19], robustness [CKM TAC'18, TGJP CDC'17, GPK RAL'17]



The lazy shepherd problem

Robust shepherd



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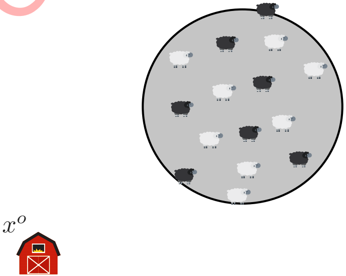

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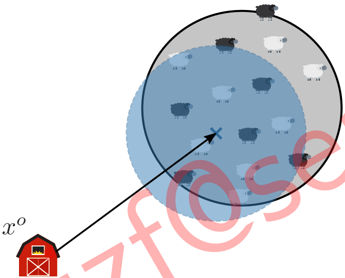

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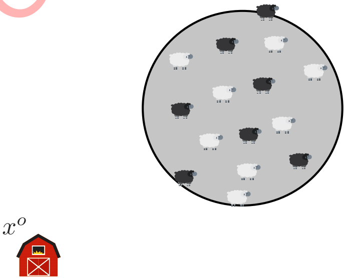

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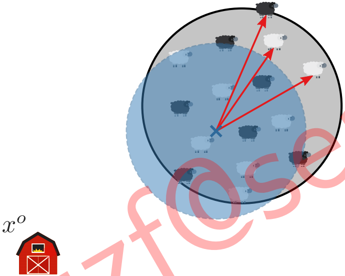

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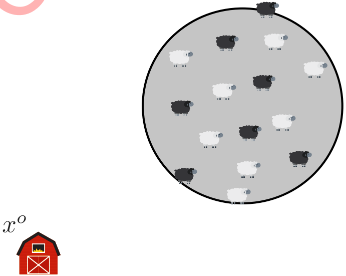

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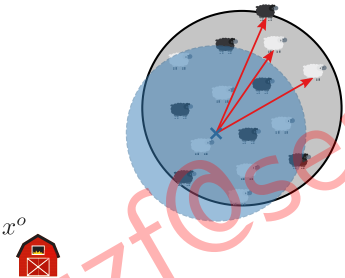

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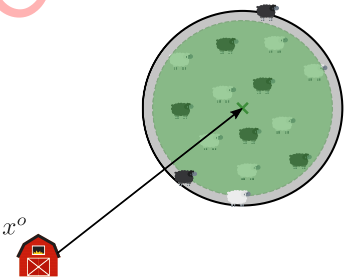

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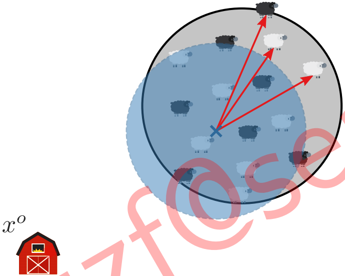

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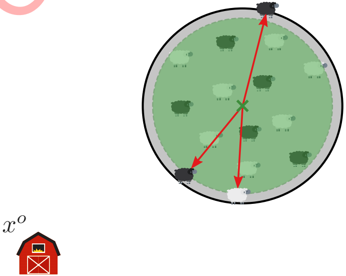

Uncertainty in optimal control

Problem (LQR with disturbances)

$$
\begin{aligned}
P^\star = \min_{\boldsymbol{x}_k, \boldsymbol{u}_k} \quad & \boldsymbol{x}_N^T \boldsymbol{P} \boldsymbol{x}_N + \sum_{k=0}^{N-1} \boldsymbol{x}_k^T \boldsymbol{Q} \boldsymbol{x}_k + \boldsymbol{u}_k^T \boldsymbol{R} \boldsymbol{u}_k \\
\text{subject to} \quad & |\boldsymbol{x}_k| \leq \bar{\boldsymbol{x}}, \quad |\boldsymbol{u}_k| \leq \bar{\boldsymbol{u}} \\
& \boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k
\end{aligned}
$$

Uncertainty in optimal control

Problem (LQR with disturbances)

$$\begin{aligned} P^\star(\Xi) = \min_{\boldsymbol{x}_k,\, \boldsymbol{u}_k} \quad & \boldsymbol{x}_N^T \boldsymbol{P} \boldsymbol{x}_N + \sum_{k=0}^{N-1} \boldsymbol{x}_k^T \boldsymbol{Q} \boldsymbol{x}_k + \boldsymbol{u}_k^T \boldsymbol{R} \boldsymbol{u}_k \\ \text{subject to} \quad & |\boldsymbol{x}_k| \leq \bar{\boldsymbol{x}}, \quad |\boldsymbol{u}_k| \leq \bar{\boldsymbol{u}} \\ & \boldsymbol{x}_{k+1} = \boldsymbol{A} \boldsymbol{x}_k + \boldsymbol{B} \boldsymbol{u}_k \end{aligned}$$

- Ξ is a random variable describing the disturbances

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- $\boldsymbol{\Xi}$ is a random variable describing the disturbances

Uncertainty in optimal control

Problem (Prototypical control with disturbances)

$$P^\star(\Xi) = \min_{\boldsymbol{z} \in \mathbb{R}^p} \quad J(\boldsymbol{z})$$

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{z}, \Xi) \preceq \boldsymbol{0}$$

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- J is a control performance measure
- $\boldsymbol{g}(\cdot, \xi)$ describes the control requirements under the disturbance ξ

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Robust optimal control

Goal

Find a deterministic $\boldsymbol{z}^\dagger$ *that is feasible for most (if not all) realizations* $\boldsymbol{\xi}$ *and whose performance* $J(\boldsymbol{z}^\dagger) \approx P^\star(\boldsymbol{\xi})$.

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$$\text{subject to} \quad \Pr\left[\boldsymbol{g}(\boldsymbol{z}, \boldsymbol{\Xi}) \preceq \mathbf{0}\right] \geq 1 - \delta$$

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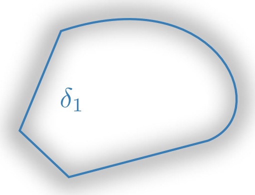

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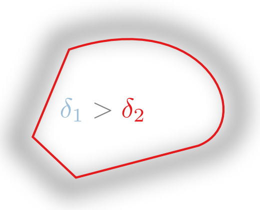

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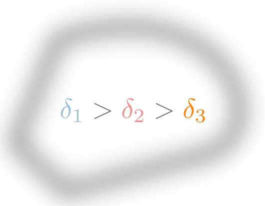

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P^\star_{\mathit{Re}}(\boldsymbol{s}) = \min_{\boldsymbol{z}\in\mathbb{R}^p} \quad & J(\boldsymbol{z}) \\
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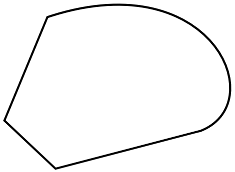

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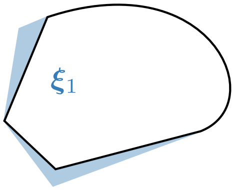

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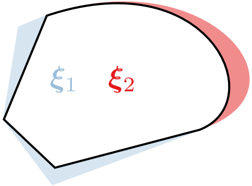

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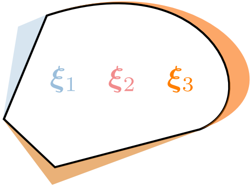

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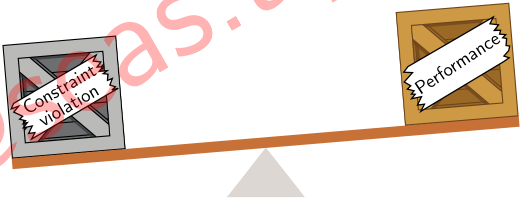

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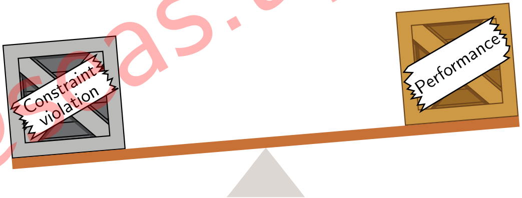

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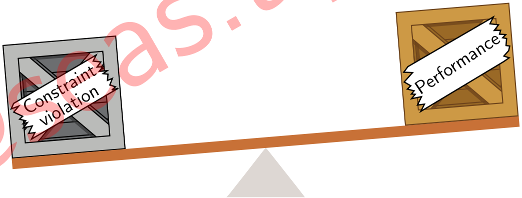

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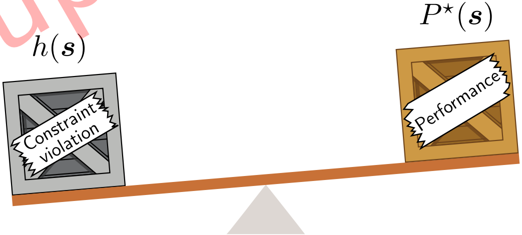
- Resilience cost: $h(\boldsymbol{s})$



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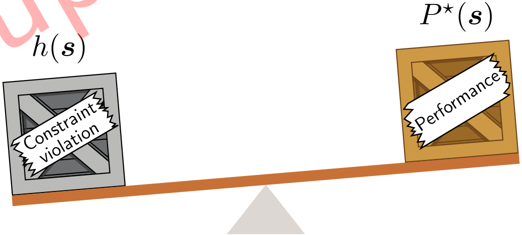
$$\nabla P^\star_{\text{Re}}(s)\big|_{s^\star,\xi} = -\nabla h(s^\star(\xi)) f_\Xi(\xi)$$

Trade-off



Resilience-by-compromise

Problem (Resilience-by-compromise)

$$P^\star_{\textsf{Re}} = \min_{\boldsymbol{z} \in \mathbb{R}^p} \quad J(\boldsymbol{z})$$

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{z}, \boldsymbol{\xi}) \leq \boldsymbol{s}^\star(\boldsymbol{\xi})$$

for $\nabla P^\star_{\textsf{Re}}(\boldsymbol{s})\Big|_{\boldsymbol{s}^\star,\, \boldsymbol{\xi}} = -\nabla h(\boldsymbol{s}^\star(\boldsymbol{\xi})) f_{\boldsymbol{\Xi}}(\boldsymbol{\xi})$

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Questions

What are the effects of disturbances?

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Is there a practical way to find $s^\star$?

Proposition

Let $\left(z^\star_{Re}, \boldsymbol{\lambda}^\star\right)$ be a primal-dual pair of the resilience-by-compromise control problem. Then,

$$\boldsymbol{s}^\star = (\nabla h)^{-1}\left[\frac{\boldsymbol{\lambda}^\star(\boldsymbol{s}^\star)}{f_{\boldsymbol{\Xi}}}\right].$$

Depends on. . .

requirement difficulty ($\lambda^\star$) disturbance likelihood (f_Ξ) resilience cost (h)

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Let $\left(z^\star_{Re}, \boldsymbol{\lambda}^\star\right)$ *be a primal-dual pair of the resilience-by-compromise control problem. Then,*

$$s^\star = (\nabla h)^{-1}\left[\frac{\boldsymbol{\lambda}^\star(s^\star)}{f_{\Xi}}\right].$$

Depends on. . .

requirement difficulty $(\boldsymbol{\lambda}^\star)$ disturbance likelihood (f_{Ξ}) resilience cost (h)

The effect of disturbances

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Linear cost: $h(\boldsymbol{s}) = \boldsymbol{\gamma}^T\boldsymbol{s} \quad \Rightarrow \quad [\boldsymbol{s}^\star]_i = [\gamma]_i^{-1}$

Quadratic cost: $h(\boldsymbol{s}) = \boldsymbol{s}^T\boldsymbol{\Gamma}\boldsymbol{s} \quad \Rightarrow \quad \boldsymbol{s}^\star = \dfrac{\boldsymbol{\Gamma}^{-1}\boldsymbol{\lambda}^\star}{f_{\boldsymbol{\Xi}}}$

Resilience-by-compromise

Problem (Resilience-by-compromise)

$$P^\star_{\textsf{Re}} = \min_{\boldsymbol{z} \in \mathbb{R}^p} \quad J(\boldsymbol{z})$$

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{z}, \boldsymbol{\xi}) \leq \boldsymbol{s}^\star(\boldsymbol{\xi})$$

for $\quad \nabla P^\star_{\textsf{Re}}(\boldsymbol{s})\Big|_{\boldsymbol{s}^\star, \boldsymbol{\xi}} = -\nabla h(\boldsymbol{s}^\star(\boldsymbol{\xi})) f_{\boldsymbol{\Xi}}(\boldsymbol{\xi})$

Questions

What are the effects of disturbances? **Depends on the requirement difficulty**

What (if any) is the relation with robustness?

Is there a practical way to find $s^\star$?

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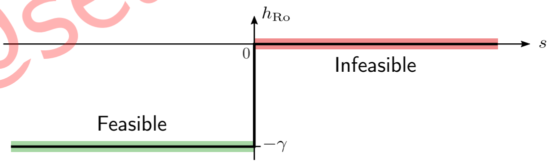
Proposition

Let $z^{\dagger}_{Re}$ be a solution the resilience-by-compromise control problem with

$$h_{\text{Ro}}(\boldsymbol{s}) = -\gamma \prod_{i=1}^{m} \mathbb{I}\left[s_i \leq 0\right].$$

For each $\gamma \geq 0$ there exists a $\delta^{\dagger}$ such that $z^{\dagger}_{Re}$ is an optimal solution of the $\delta^{\dagger}$-robust problem.



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$$P^\star_{\textit{Re}} = \min_{\boldsymbol{z}\in\mathbb{R}^p} \quad J(\boldsymbol{z})$$

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$$h(\boldsymbol{s}) = -\gamma \underbrace{\prod_{i \in \mathcal{H}} \mathbb{I}\left[s_i \leq 0\right]}_{\mathcal{H}\text{: hard (critical) requirements}}$$

Robustness vs. Resilience revisited

Problem (Resilience-by-compromise)

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$$h(\boldsymbol{s}) = \underbrace{-\gamma \prod_{i \in \mathcal{H}} \mathbb{I}\left[s_i \leq 0\right]}_{\mathcal{H}\text{: hard (critical) requirements}} + \underbrace{\sum_{i \in \mathcal{S}} h_i\left[s_i(\boldsymbol{\Xi})\right]}_{\mathcal{S}\text{: soft (nominal) requirements}}$$

Resilience-by-compromise

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Is there a practical way to find $s^\star$? **Modified Arrow-Hurwicz**

Modified Arrow-Hurwicz

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Modified Arrow-Hurwicz

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- Update the primal:

$$\boldsymbol{z}^+ = \boldsymbol{z} - \eta \left[\nabla_{\boldsymbol{z}} J(\boldsymbol{z}) - \int \boldsymbol{\lambda}(\boldsymbol{\xi})^T \nabla_{\boldsymbol{z}} g(\boldsymbol{z}, \boldsymbol{\xi}) d\boldsymbol{\xi} \right]$$

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- Update the dual:

$$\boldsymbol{\lambda}^+(\boldsymbol{\xi}) = \Pi_{R^m_+}\left[\boldsymbol{\lambda}(\boldsymbol{\xi}) + \eta\Big(\boldsymbol{g}(\boldsymbol{z},\boldsymbol{\xi}) - \boldsymbol{s}(\boldsymbol{\xi})\ \Big)\right]$$

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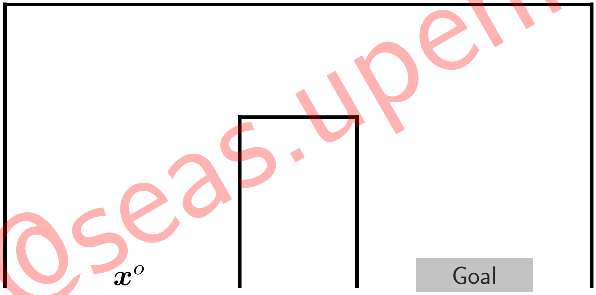
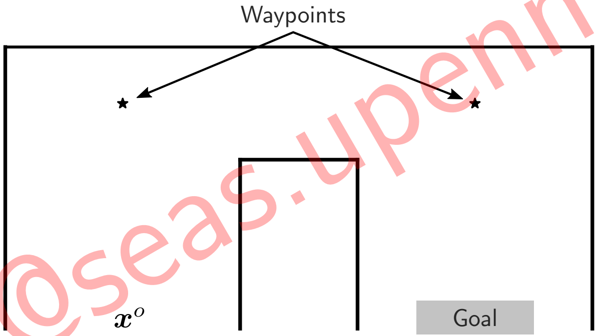
- Update the dual for resilient slacks:

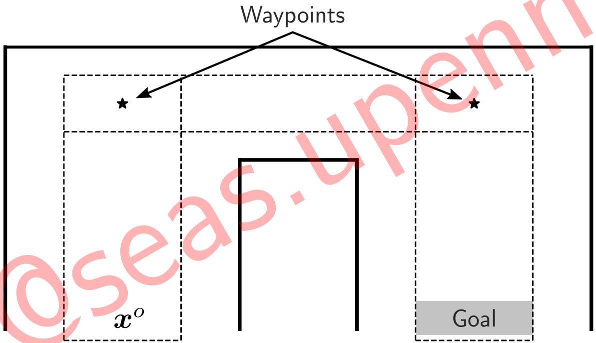
$$\boldsymbol{\lambda}^+(\boldsymbol{\xi}) = \Pi_{R^m_+}\left[\boldsymbol{\lambda}(\boldsymbol{\xi}) + \eta\left(\boldsymbol{g}(\boldsymbol{z},\boldsymbol{\xi}) - (\nabla h)^{-1}\left[\frac{\boldsymbol{\lambda}(\boldsymbol{\xi})}{f_{\boldsymbol{\Xi}}(\boldsymbol{\xi})}\right]\right)\right]$$

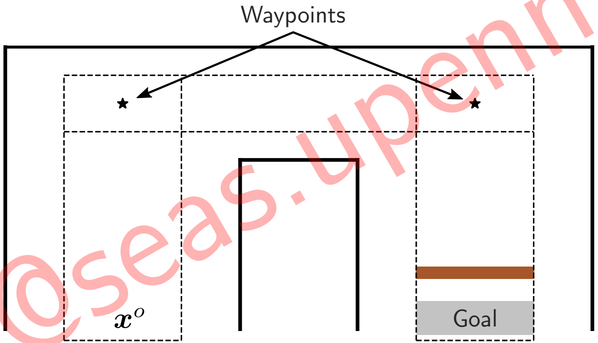
Waypoint navigation



Waypoint navigation




Waypoints
x^o
Goal


Waypoints
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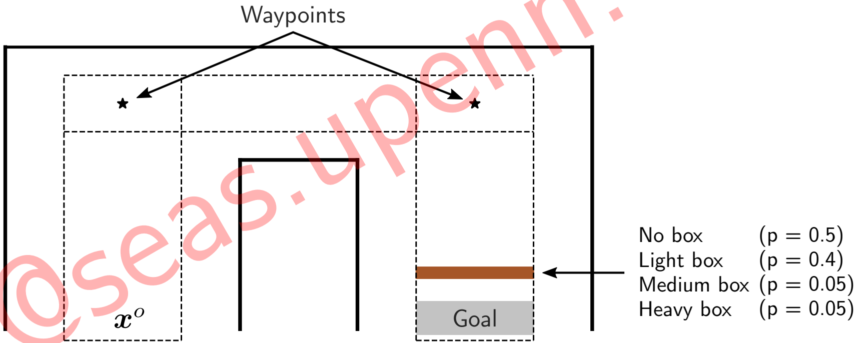
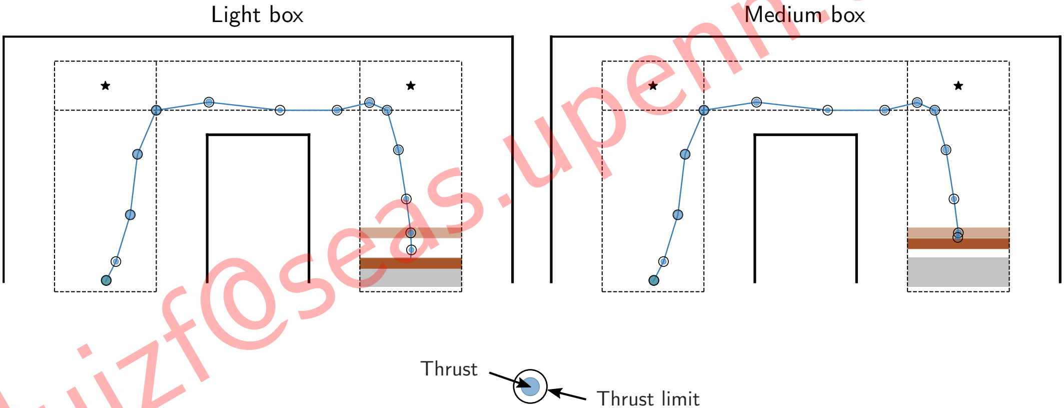
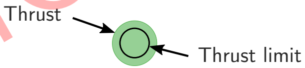
Waypoint navigation




Light box
Medium box
Thrust
Thrust limit

Resilient waypoint navigation

Resilient waypoint navigation



Resilient waypoint navigation

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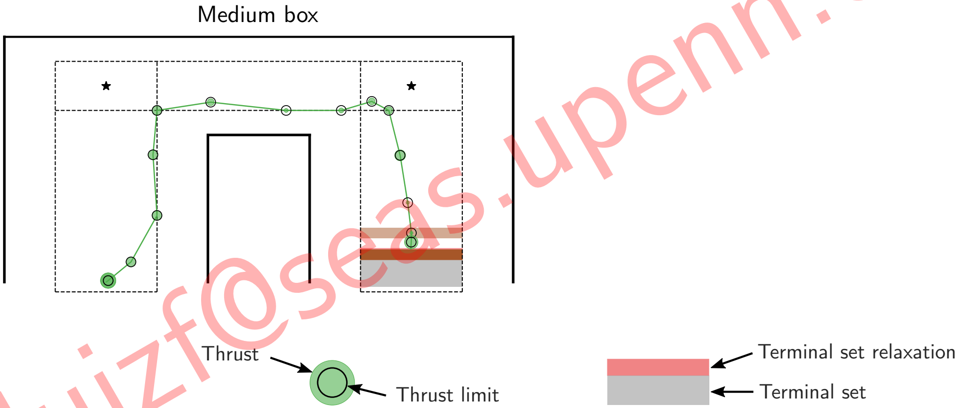
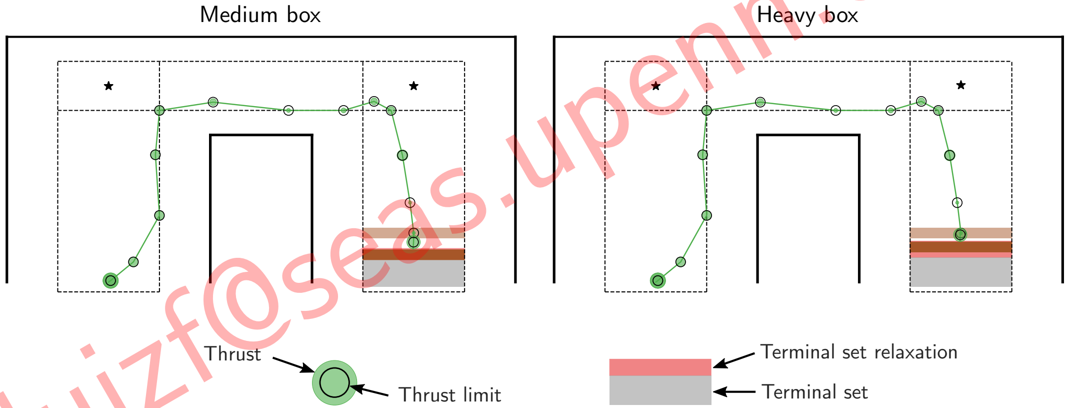


Resilient waypoint navigation



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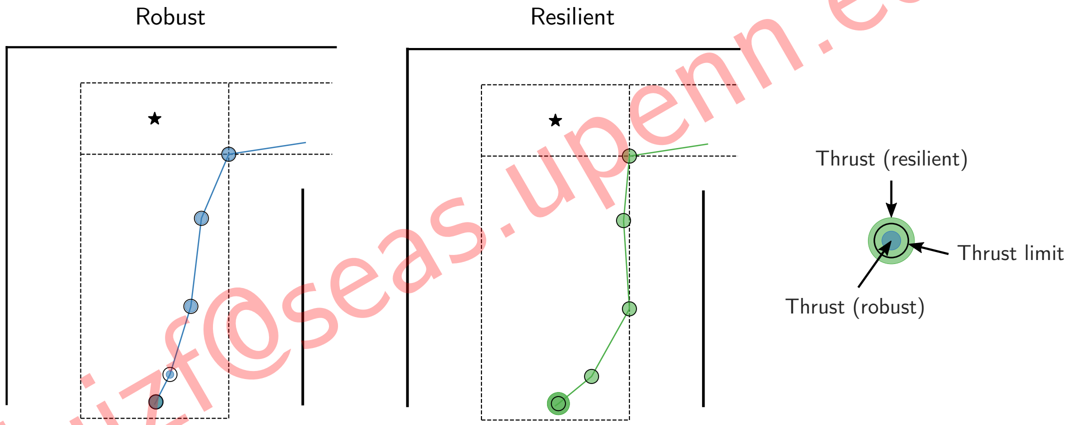

Conclusion

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  - In the paper: online resilience using MPC

RESILIENT CONTROL: COMPROMISING TO ADAPT

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https://arxiv.org/abs/2004.03726

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