COMBINATION OF ADAPTIVE FILTERS FOR RELATIVE NAVIGATION

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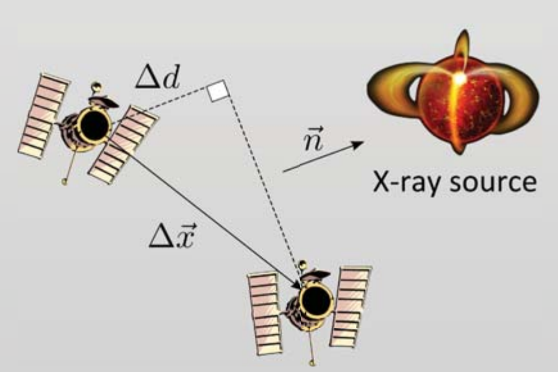
ABSTRACT

Relative navigation of spacecrafts may be accomplished via time delay estimates. In this work an adaptive filtering approach is employed, which involves an estimation and a detection step. By formally posing a detection problem, a more meaningful detector, that embeds a reliability measure into the delay estimates, is proposed. The estimation step is enhanced via convex combination schemes, that address the Poisson distributed signals, sparse channel and low signal-to-noise ratio. To evaluate time delay estimation techniques, different criteria based on probability of detection are studied, leading to a new figure of merit. The resulting solution outperforms the existing adaptive filters techniques under the new criterion, as shown by simulations.

INTRODUCTION

- Relative Navigation
  - The use of signals delay is a well established method for navigation (LORAN, GPS…)
  - In space, however, access to these beacons becomes more intricate (e.g. deep space probes operate beyond GPS range).
  - Since many applications (e.g. interferometric imaging) only require relative positioning, celestial X-ray sources have often been proposed as bearing signals.



Relative Navigation = Time Delay Estimation (TDE)

$$\Delta d = \vec{n} \cdot \Delta\vec{x} = c t_d$$

- $\Delta\vec{x}$ is the relative position vector;
- $\vec{n}$ is the normal vector;
- Δd is the relative distance in the direction $\vec{n}$;
- t_d is the delay between the received signals;
- c is the speed of light.

If more sources are used, three dimensional positions can be calculated [1,2].

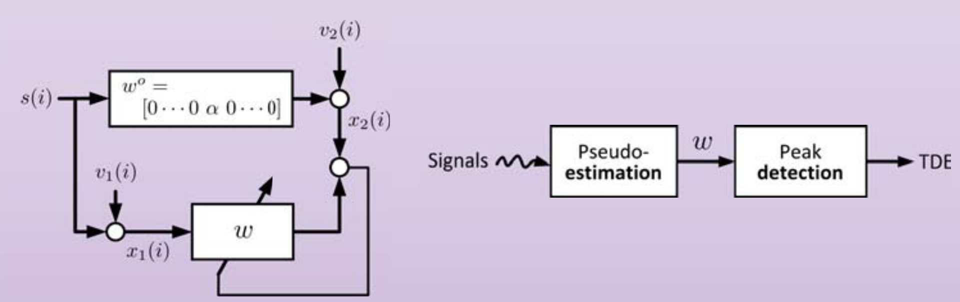
THE TDE PROBLEM

- The signal model

$$x_1(i) = s(i) + v_1(i)$$
$$x_2(i) = \alpha s(i - n_d) + v_2(i)$$

where $s(i)$ is a Poisson distributed measurement of the X-ray source, $0 < \alpha < 1$ is an attenuation factor, $v_1(i)$ and $v_2(i)$ are independent noises, $n_d = \lfloor t_d / t_s \rfloor$, and t_s is the sampling period [1,2].

- The adaptive filtering (AF) solution



✓ Computationally simple, robust and model-free [3];
✓ May, under certain conditions, be asymptotically efficient [4,5];
✓ Less sensitive to changes in signal spectra than GCC [4,6].

THE DETECTION PROBLEM

- The classical detector [2,4,6,7]

$$\hat{t}_d = \operatorname{argmax}(w) \cdot t_s$$

✗ Fails to address the reliability of the detected peak.

- Detection and false alarm

$$H^1: \quad w = w^o + \tilde{w} \quad \text{(delay in the signals)}$$
$$H^0: \quad w = \tilde{w} \quad \text{(noise)}$$

$\tilde{w} = \mathcal{N}(\Psi, R_{\tilde{w}}) \rightarrow$ corruption in the pseudo-estimate where $\Psi = \psi col\{1\}$ and $R_{\tilde{w}} = E\tilde{w}\tilde{w}^*$

$$\Lambda = \frac{f(\hat{w}|H^1)}{f(\hat{w}|H^0)} \underset{H^0}{\overset{H^1}{\gtrless}} \eta$$
$$w^{oT} R_{\tilde{w}}^{-1} (\hat{w} - \Psi) \underset{H^0}{\overset{H^1}{\gtrless}} \ln\eta + \tfrac{1}{2} w^{oT} R_{\tilde{w}}^{-1} w^o \triangleq \nu$$

Log-likelihood ratio test

Probability of false alarm

$$P_F = P\left[H^1|H^0\right] = P\left[w^{oT} R_{\tilde{w}}^{-1}(\tilde{w} - \Psi) > \nu\right]$$

Probability of detection

$$P_D = P\left[H^1|H^1\right] = P\left[w^{oT} R_{\tilde{w}}^{-1}(w^o - \Psi) > \nu\right]$$

$$\Rightarrow P_D = Q\left(Q^{-1}(P_F) - \frac{w^{oT} R_{\tilde{w}}^{-1}(w^o - \Psi)}{(w^{oT} R_{\tilde{w}}^{-1} w^o)^{1/2}}\right)$$

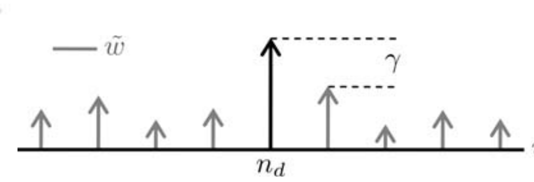
$$\frac{\alpha - \psi}{r} \qquad r = [R_{\tilde{w}}]^{1/2}_{n_d, n_d}$$

✓ Induces a reliability measure

- The new time delay detector

$$\frac{\alpha - \psi}{r} \sim \frac{\max(w) - \max(w')}{\max(w')}$$

where w' is the $(M-1)$ x 1 vector capturing all the weights in w but $w[n_d]$



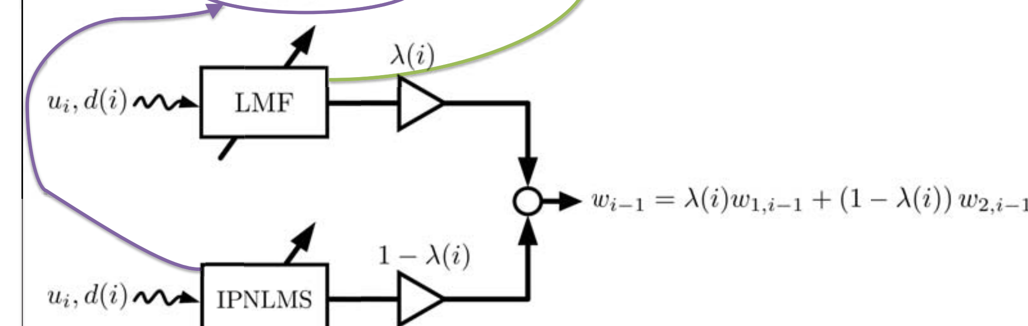
$$\hat{t}_d = \begin{cases} \operatorname{argmax}(w) \cdot t_s, & \frac{\max(w) - \max(w')}{\max(w')} > \gamma \\ undefined, & \text{otherwise} \end{cases}$$

THE ESTIMATION PROBLEM

- AF to improve detection
  - A huge class of AFs attempts to minimize $\text{MSE} = \text{E}\|d(i) - u_i w\|_2^2$, with $u_i \triangleq [\, x_1(i) \cdots x_1(i - M + 1) \,]$ and $d(i) \triangleq x_2(i)$ [3].

$$\min \text{MSE} \Rightarrow \min Tr(R_{\tilde{w}}) \Rightarrow \min r \Rightarrow \max P_D$$

- Convex combination [8-10]
  - Application characteristics: Poisson-distributed signals, low SNR and sparse channel



$$w_{1,i} = w_{1,i-1} + \mu u_i^* e_1^3(i) \quad \text{(LMF)}$$
$$w_{2,i} = w_{2,i-1} + \frac{\mu G}{\epsilon + u_i G u_i^*} u_i^* e_2(i) \quad \text{(IPNLMS)}$$

$$e_k(i) = d(i) - u_i w_{k,i-1}$$
$$G = diag\left\{\frac{1-\beta}{2M} col\{1\} + (1+\beta)\frac{|w_{2,i-1}|}{\|w_{2,i-1}\|_1}\right\}$$
$$-1 \le \beta \le 1$$

$$\lambda(i) = \frac{1}{1 + e^{-a(i-1)}} \qquad a(i) = a(i-1) + \mu_a e(i) u_i^* (w_{1,i-1} - w_{2,i-1}) \lambda(i)(1 - \lambda(i))$$

A NEW FIGURE OF MERIT: DISCRIMINATION

- Existing detection criteria
  - Average weight [1,2]: $\Pr\left[\operatorname{argmax}(\text{E}w) = n_d\right]$
    ✗ Misleading at low SNRs
  - Accuracy percentage (ML) [11]: [Correct detections]/[Number of tests]
    ✗ Only works for the classical detector
- A more convenient figure of merit
  - For the new detector:

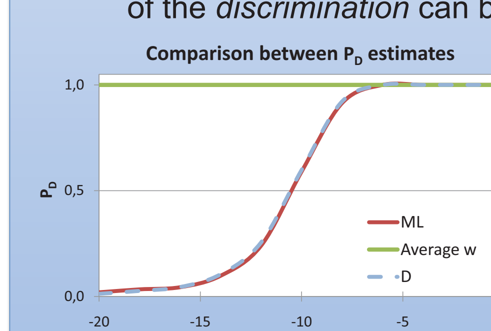
$$P_D = \Pr\left(\frac{\max(w) - \max(w')}{\max(w')} > \gamma \,\middle|\, \operatorname{argmax}(w) = n_d\right)$$

  - Noting that $\operatorname{argmax}(w) = n_d \Leftrightarrow \max(w) = \hat{w}[n_d]$ we define

$$\mathcal{D} = \frac{\hat{w}[n_d]}{\max(w')} \quad \text{(discrimination)}$$

so that $P_D = \Pr[\mathcal{D} > \gamma + 1]$

  - With one Monte Carlo run, the average ($\mu_{\mathcal{D}}$) and variance ($\sigma_{\mathcal{D}}^2$) of the *discrimination* can be used to evaluate P_D



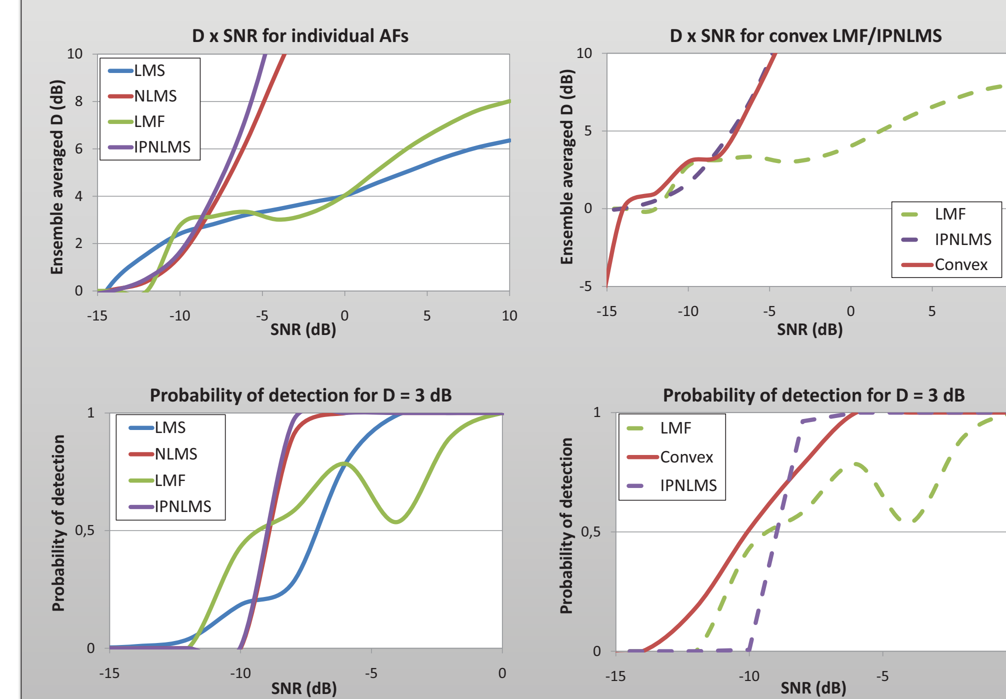
$$\hat{P}_D^{\mathcal{D}} = Q\left(\frac{\gamma + 1 - \mu_{\mathcal{D}}}{\sigma_{\mathcal{D}}}\right)$$

✓ Different scenarios (γ) can be assessed with one simulation
✓ Gives a way to design γ to get a given P_D

SIMULATIONS

Experimental setup

$\alpha = 0.9$, $M = 100$, the AF order, $n_d = 50$, $\theta_s = 1$, the variance of the source signal, $\theta_{v_1} = \theta_{v_2}$, the measurement noise variances, $\mu_{LMS} = 1.2 \cdot 10^{-4}/M$, $\mu_{NLMS} = 10^{-2}$, $\mu_{LMF} = 9 \cdot 10^{-6}/M$, $\mu_{IPNLMS} = 10^{-2}$, $\beta = -0.5$, $\epsilon = 10^{-6}$ and $\mu_a = 0.5$



CONCLUSION

TDE problems based on AFs involve an estimation stage followed by a detection stage. Both steps were addressed in this work by proposing a new detector that embeds a practical reliability measure and a convex combination scheme that improves the probability of detection. The latter was argued as a more meaningful metric to evaluate TDE solutions. Future works will include the use of hierarchical combinations [25] to employ other PNLMS-based AFs [26] and the study of non-stationary delays.

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