TRANSIENT PERFORMANCE OF AN INCREMENTAL COMBINATION OF LMS FILTERS

Polytechnic School
Dept. of Electronics Systems
São Paulo — Brazil

Luiz F. O. Chamon
chamon@usp.br

Cássio G. Lopes
cassio@lps.usp.br

ABSTRACT

Incremental combinations were first introduced as a solution to the convergence stagnation issue in parallel-independent combinations. Since then, this topology has been shown to enhance performance to the point of a combination of LMS filters outperforming the APA with lower computational complexity. In order to better understand and improve this structure, the present work develops mean and mean-square transient models for the incremental combination of two LMS filters, that is shown to be a generalization of the data reuse LMS (DR-LMS). By formulating the optimal supervisor and deriving its constraints, the previously proposed adaptive combiner is redesigned to improve the combination's overall performance.

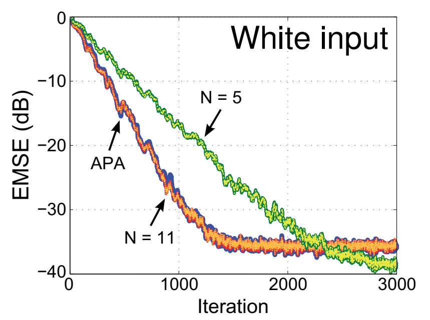
INTRODUCTION

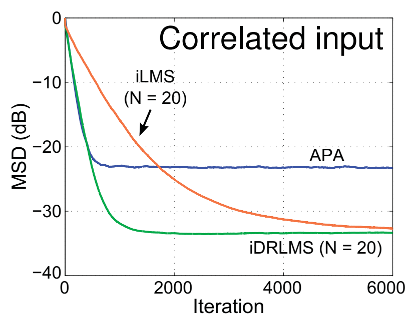
Combination of AFs:

- **Definition:** set of AFs combined by a supervisor.
- Used when the accurate design of a single filter is difficult or the resulting algorithm's complexity is too high
- (Parallel) Independent components ⇒ convergence stagnation
  - Solutions: transfer of coefficients, coefficients feedback, **incremental combination**

Incremental combination:

- Fast convergence
- Supervisor design remains an open question





COMBINATION OF ADAPTIVE FILTERS

Adaptive filters

LMS

$$w_{n,i} = w_{n,i-1} + \mu_n u_{n,i}^T e_n(i)$$

$w_{n,i} \to M \times 1$ coefficient vector of the n^{th} component at iteration i

$\mu_n \to n^{th}$ component step size

$e_n(i) = d_n(i) - u_{n,i} w_{n,i-1} \to$ output estimation error

$u_i \to 1 \times M$ input regressor—$\operatorname{E} u(i)^2 = \sigma_u^2$

$d(i) = u_i w^o + v(i) \to$ desired signal

$w^o \to M \times 1$ vector that models the unknown system

$v(i) \to$ i.i.d. measurement noise—$\operatorname{E} v(i)^2 = \sigma_v^2$

Data reuse

$$\{u_{n,i}, d_n(i)\} = \{u_{i-n+1}, d(i-n+1)\} \qquad \textit{(data buffering)}$$

$$\{u_{n,i}, d_n(i)\} = \{u_i, d(i)\} \qquad \textit{(data sharing)}$$

Parallel combination with coefficients feedback

$$w_{n,i-1} = \delta(i - rL)\, w_{i-1} + (1 - \delta(i - rL))\, w_{n,i-1}$$
$$w_{n,i} = w_{n,i-1} + \mu_n u_i^* \left[d(i) - u_i w_{n,i-1}\right]$$
$$w_i = \sum_{n=1}^{N} \eta_n(i)\, w_{n,i}$$

Incremental combination

$$w_{0,i} = w_{i-1}$$
$$w_{n,i} = w_{n-1,i} + \eta_n(i) \mu_n u_i^* [d(i) - u_i w_{n-1,i}]$$
$$w_i = w_{N,i}$$

MEAN PERFORMANCE

Derivations are carried on for $N = 2$ LMS filters assuming $u(i)$ arises from a real zero-mean i.i.d. process.

Global coefficients error recursion

$$\tilde{w}_i = \tilde{w}_{i-1} - \left[\bar{\mu}(i) - \mu'(i)\, \|u_i\|^2\right] u_i^T e(i)$$

$$\tilde{w}_i = w^o - w_i \qquad e_a(i) = u_i \tilde{w}_{i-1}$$

$$\bar{\mu}(i) = \eta_1(i)\mu_1 + \eta_2(i)\mu_2 \qquad \mu'(i) = \eta_1(i)\eta_2(i)\mu_1\mu_2$$

A.1 (*Data independence assumptions*) $\{u_i\}$ is i.i.d. and independent of $v(j), \forall i, j$. Therefore, $\{u_i, \tilde{w}_j\}$, $\{d(i), d(j)\}$, $\{u_i, d(j)\}$ are independent for $i > j$.

A.2 (*Supervisor separation principle*) $\operatorname{E}[\eta_n(i) u_i] \approx \operatorname{E}\eta_n(i) \operatorname{E} u_i$ and $\operatorname{E}[\eta_n(i) e_a(i)] \approx \operatorname{E}\eta_n(i) \operatorname{E} e_a(i)$

Mean coefficients error recursion

$$\operatorname{E}\tilde{w}_i = [1 - \operatorname{E}\bar{\mu}(i)\sigma_u^2 + \operatorname{E}\mu'(i)(M+2)\sigma_u^4]\operatorname{E}\tilde{w}_{i-1}$$

MEAN-SQUARE PERFORMANCE

$$\text{MSD}(i) = \operatorname{E}\|\tilde{w}_{i-1}\|^2 = \operatorname{Tr}(K_i)$$

$$\text{EMSE}(i) = \operatorname{E}\|e_a(i)\|^2 = \operatorname{Tr}(R_u K_i) = \sigma_u^2 \text{MSD}(i)$$

$$\text{MSE}(i) = \operatorname{E}\|e(i)\|^2 = \operatorname{E}\|e_a(i) + v(i)\|^2 = \text{EMSE}(i) + \sigma_v^2$$

A.3 (*Gaussian data*) u_i is a Gaussian vector

Mean-square coefficients error recursion

$$\text{MSD}(i+1) = A \cdot \text{MSD}(i) + b \cdot \operatorname{Tr}(R_u)\sigma_v^2$$

$$A = 1 - 2\operatorname{E}\bar{\mu}\sigma_u^2 + 2(M+2)\operatorname{E}\mu'\sigma_u^4 + (M+2)\beta$$

$$b = \operatorname{E}\bar{\mu}^2 - 2\operatorname{E}\bar{\mu}\mu'(M+2)\sigma_u^2 + \operatorname{E}\|\mu'\|^2 (M+2)(M+4)\sigma_u^4$$

A.4 (*Small step sizes*) Higher order powers of μ_n are negligible—i.e., $[\operatorname{Tr}(R_u)\mu_n]^\ell \approx 0, \forall \ell > 2$.

$$A' = 1 - 2\operatorname{E}\bar{\mu}\sigma_u^2 + (M+2)(\operatorname{E}\bar{\mu}^2 + 2\operatorname{E}\mu')\sigma_u^4$$
$$b' = \operatorname{E}\bar{\mu}^2$$

SUPERVISOR ANALYSIS

Optimal supervisor

$$\nabla_{\eta_1,\eta_2} \text{MSD}(i+1) = 0 \Leftrightarrow \eta_1\mu_1 = \eta_2\mu_2$$

$$\eta_n^o(i) = \frac{1}{\mu_n} \frac{\text{MSD}(i)}{3(M+2)\sigma_u^2 \text{MSD}(i) + 2M\sigma_v^2}$$

$$n = 1, 2$$

Supervisor constraint

$$\lim_{i\to\infty} \operatorname{E}\tilde{w}_i = 0 \Leftrightarrow \left|1 - \operatorname{E}\bar{\mu}(i)\sigma_u^2 + \operatorname{E}\mu'(i)(M+2)\sigma_u^4\right| < 1$$

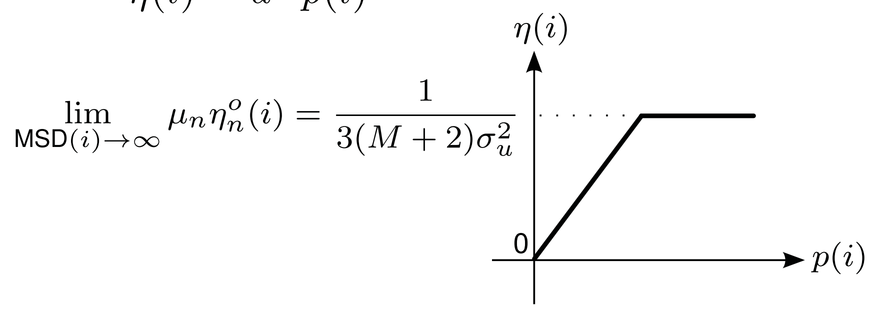
$$0 < \operatorname{E}\eta_n(i)\mu_n < \frac{2}{(M+2)\sigma_u^2}$$

Adaptive supervisor

Deterministic supervisor: $\eta(i) = \dfrac{1}{1 + e^{s(i-\ell)}}$

Filtered error supervisor:

$$p(i) = \alpha\, p(i-1) + (1-\alpha)\, \|e(i)\|^2$$
$$\eta(i) = a \cdot p(i)$$



SIMULATIONS

$$\sigma_u^2 = 1 \qquad \sigma_v^2 = 10^{-3}$$
$$w^o = \text{col}\{1\}/\sqrt{M} \qquad M = 10.$$

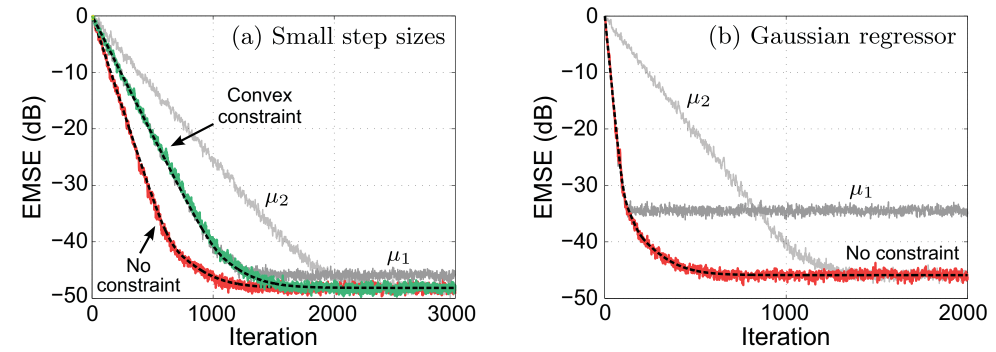
Step sizes: $\mu_1 = 0.05$ and $\mu_2 = 0.005$

Small step sizes: $\mu_1 = 0.005$ and $\mu_2 = 0.003$

Ensemble averages: 200 independent realizations.

Mean-square model validation

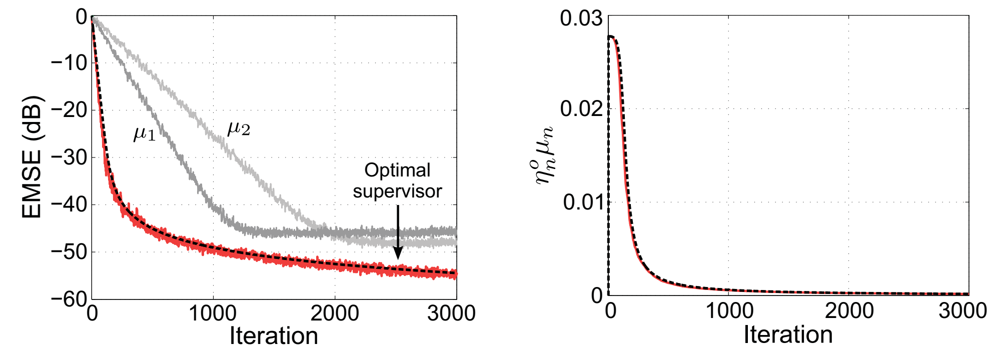
Deterministic supervisor



Convex constraint: $\eta_1(i) = \eta(i)$ and $\eta_2(i) = 1 - \eta(i)$

No constraint: $\eta_1(i) = \eta(i)$ and $\eta_2(i) = 1$

Optimal supervisor



Combination/supervisor comparison