TOWARDS SPATIALLY UNIVERSAL ADAPTIVE DIFFUSION NETWORKS

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CONTRIBUTIONS

(i) Analysis of individual node performance and spatial universality for ANs

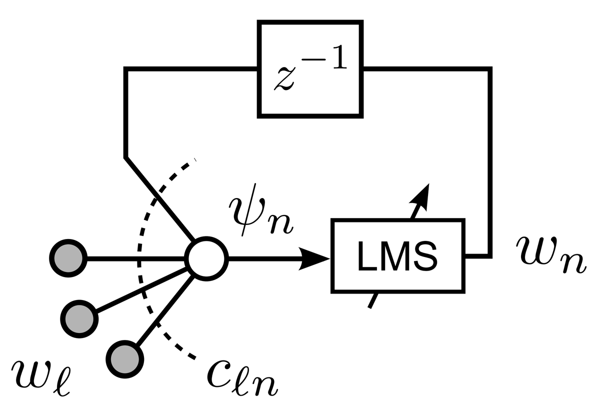
(ii) New combiner that promotes spatial universality by network and node-level feedback

(iii) Analysis of the network learning phenomenon

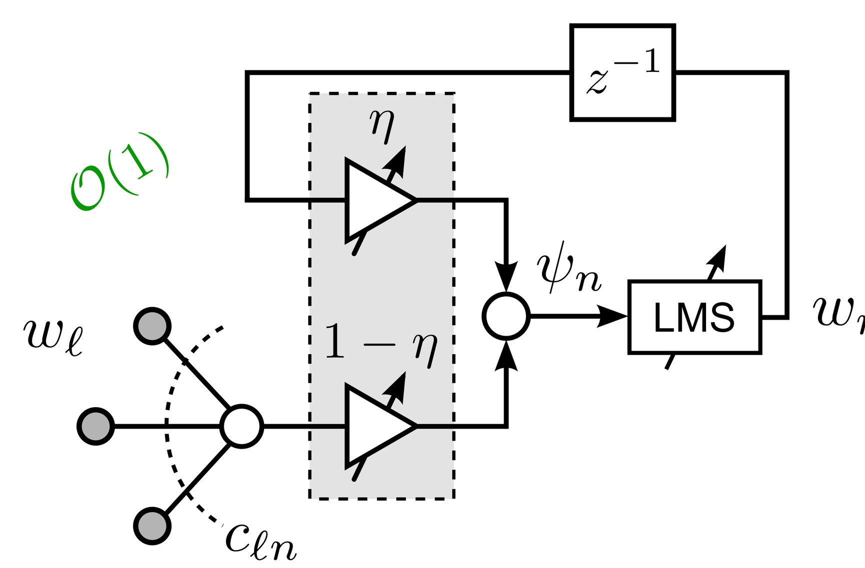
BACKGROUND

Adaptive network

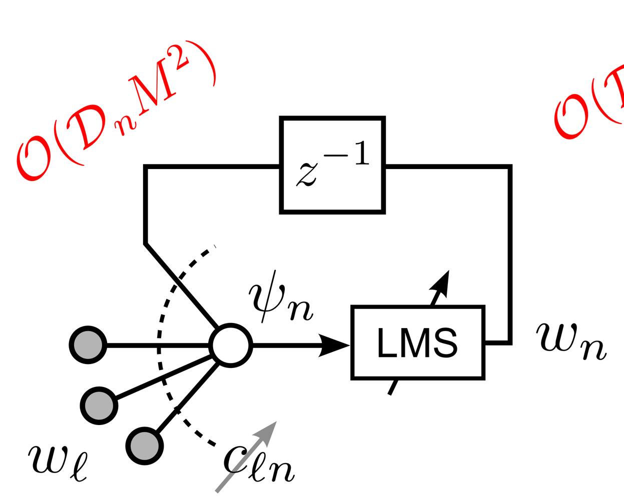
Set of agents—called *nodes*—that cooperate to estimate a set of parameters using AFs operating on local data.



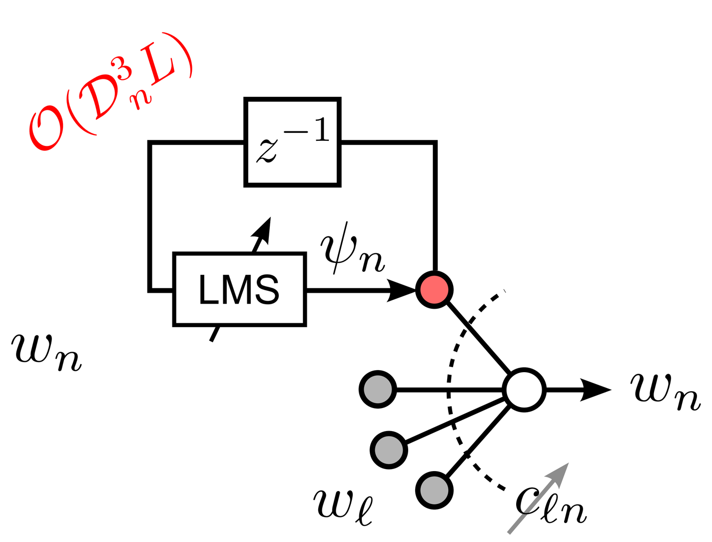
Diffusion LMS



Adaptive diffusion



MSD combiner

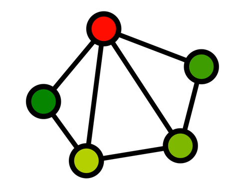


LS combiner

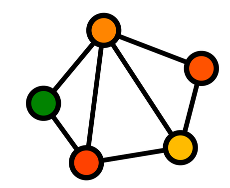
TOWARDS SPATIAL UNIVERSALITY

Desired properties of ANs

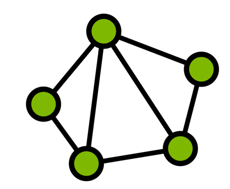
(i) Reject poor node



(ii) Exploit an exceptional node



(iii) Node performance homogeneity

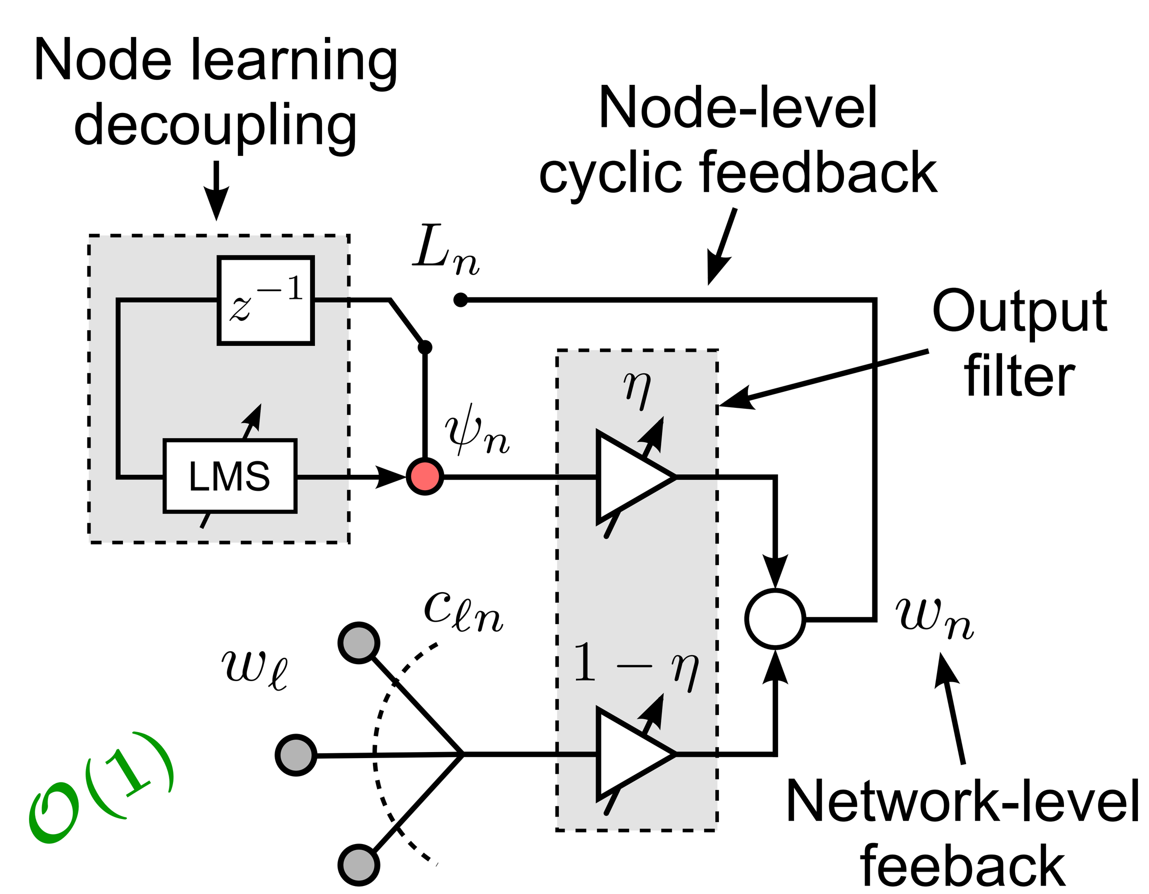


Spatial universality

Definition 1. A node is said to be *locally universal* when is at least as good as the best node in its neighborhood.

Definition 2. An AN is said to be *universal w.r.t. the non-cooperative strategy* when all its nodes perform at least as well the best non-cooperative AF in the network.

Spatial universality promoting strategy



NETWORK LEARNING

Global network recursion

$$W_i = H_i \Psi_{i-1} + (I - H_i) C^T W_{i-1}$$
$$A_i = A_{i-1} + \bar{\mathcal{M}}_{a,i} H_i (I - H_i) \mathcal{Y}_{i-1} W_i$$

$$W_i = \operatorname{col}\{w_{n,i}\} \quad \Psi_i = \operatorname{col}\{\psi_{n,i}\} \quad \bar{\mathcal{M}}_{a,i} = \operatorname{diag}\{\bar{\mu}_{a,n}\}$$
$$A_i = \operatorname{col}\{a_n(i)\} \quad H_i = \operatorname{diag}\{[1 + e^{-a_n(i-1)}]^{-1}\}$$
$$\mathcal{Y}_{i-1} = \operatorname{diag}\left\{C^T W_{i-1} - \Psi_{i-1}\right\}$$

At steady-state: $\Psi_i \sim \text{Normal}(b, R_\Psi)$

Effect of network-level feedback

Without feedback:

$$W_i = \underbrace{\left[H_i + (I - H_i) C^T\right]}_{\check{C}} \Psi_{i-1}$$

With feedback:

$$W_i = H_i \Psi_{i-1} + \sum_{k=1}^{i-1} \prod_{j=0}^{i-k-1} \left[(I - H_{i-j}) C^T\right] H_k \Psi_{k-1} + \prod_{k=0}^{i-1} \left[(I - H_{i-k}) C^T\right] \Psi_0$$

Network learning behavior

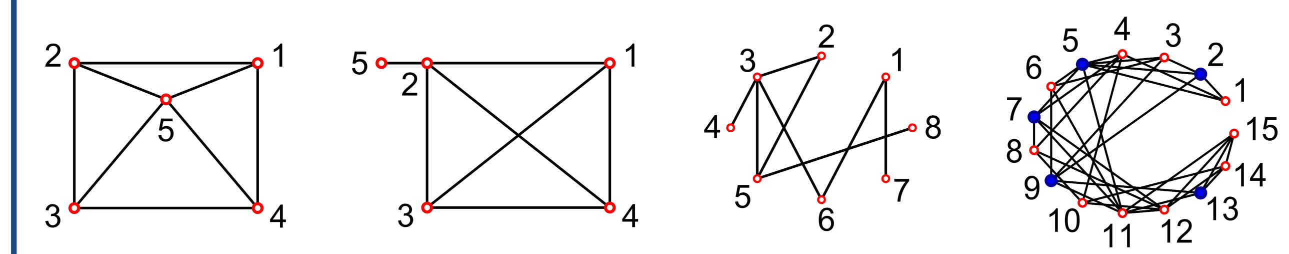
$$\bar{W}_i = \underbrace{\bar{H}_i b}_{\text{Node learning}} + \underbrace{(I - \bar{H}_i) \boldsymbol{C^T \bar{W}_{i-1}}}_{\text{Network learning}}$$

$$K_i = \bar{H}_i (R_\Psi + b b^T) \bar{H}_i + (I - \bar{H}_i) C^T \bar{W}_{i-1} b^T \bar{H}_i + \bar{H}_i b \bar{W}_{i-1}^T C (I - \bar{H}_i) + (I - \bar{H}_i) C^T K_{i-1} C (I - \bar{H}_i)$$

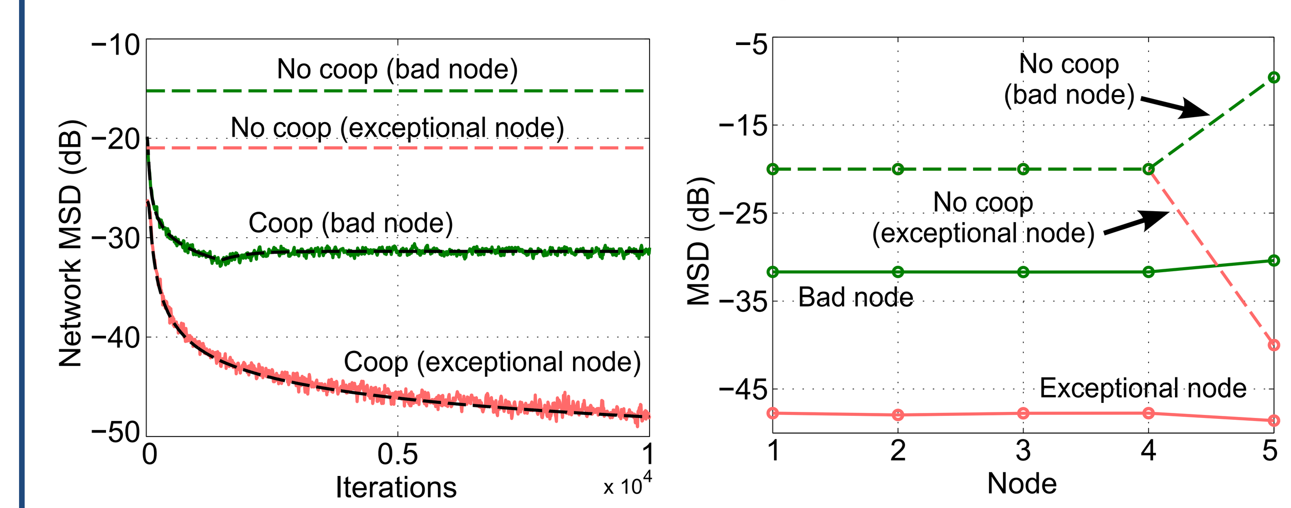
$$\bar{A}_i = \bar{A}_{i-1} + \operatorname{E} \bar{\mathcal{M}}_{a,i} \bar{H}_i (I - \bar{H}_i) \mathcal{K}_i$$

$$[\mathcal{K}_i]_n = [1 - \eta_n(i)] \underbrace{\boldsymbol{c_n^T K_{i-1} c_n}}_{\mathcal{N}\text{ performance}} + [2\eta_n(i) - 1] \underbrace{\boldsymbol{c_n^T (\bar{W}_{i-1} \circ b)}}_{\mathcal{N}\text{ bias}} - \eta_n(i) \underbrace{\boldsymbol{(\sigma_n^2 + b_n^2)}}_{\text{Node performance}}$$

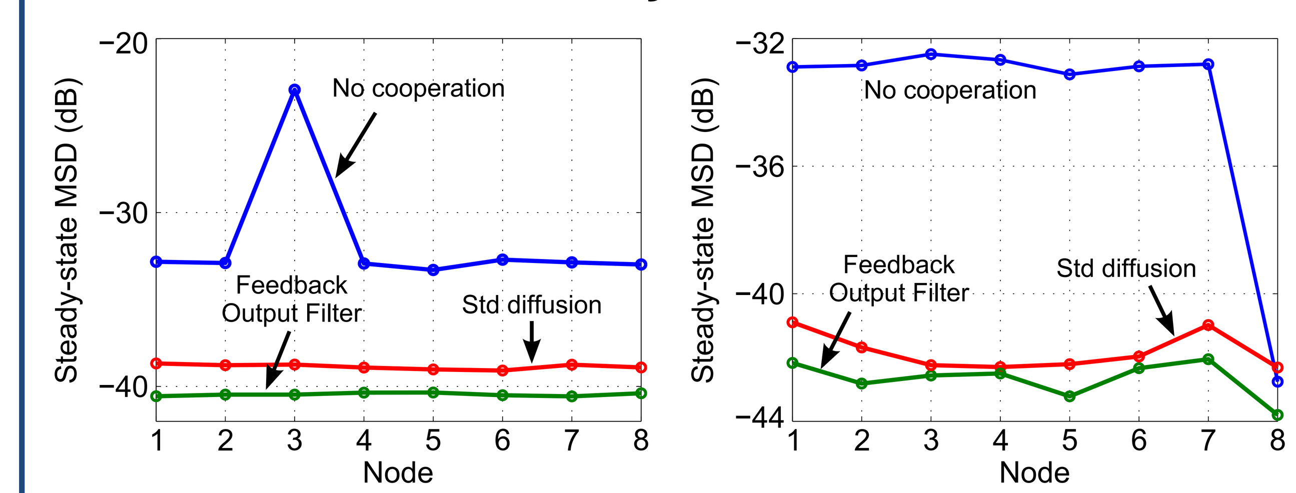
SIMULATIONS



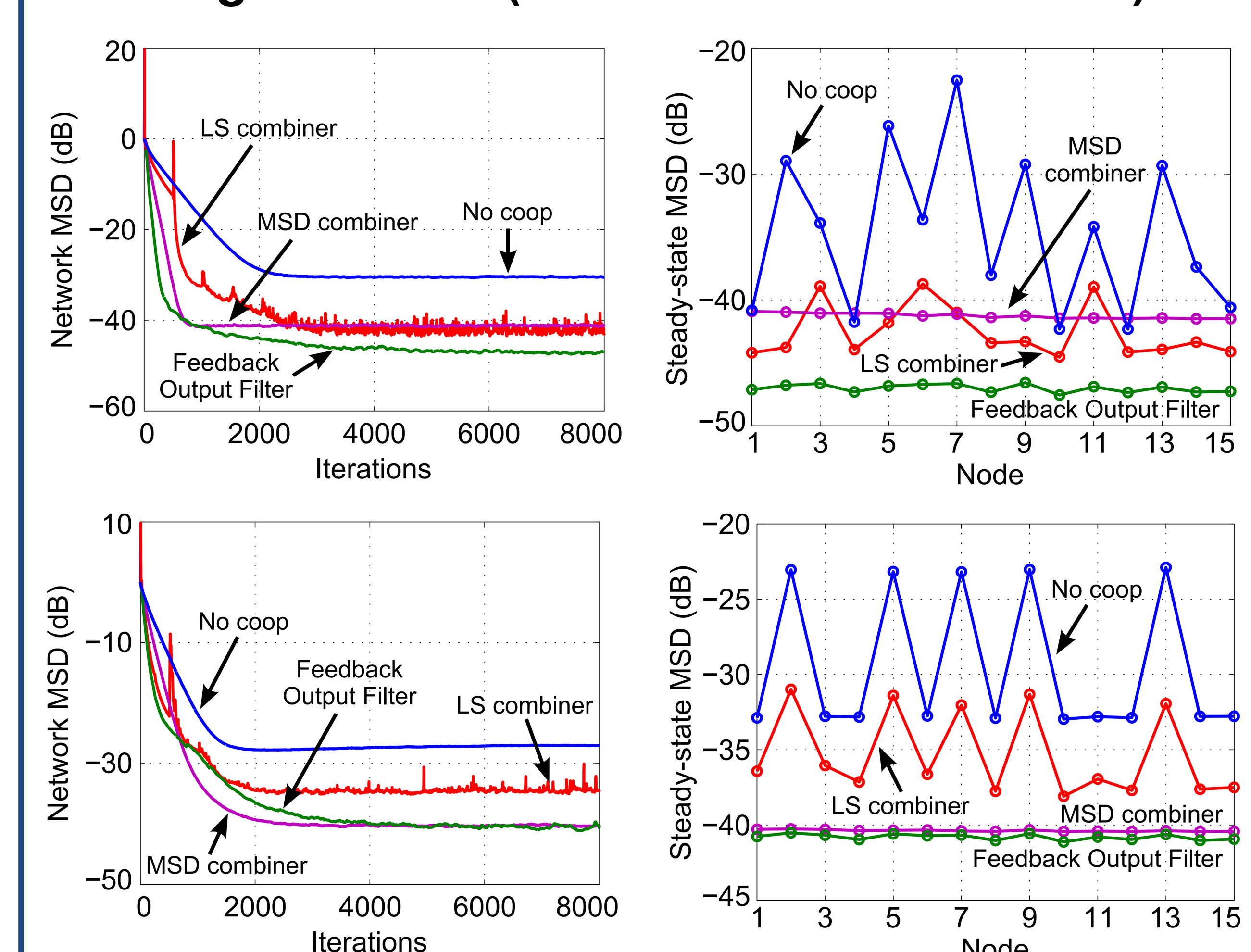
Network learning model



Arbitrary network



Large network (with node-level feedback)



ACKNOWLEDGEMENT

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