COMBINATION OF ADAPTIVE FILTERS WITH COEFFICIENTS FEEDBACK

Polytechnic School
Dept. of Electronics Systems
São Paulo – Brazil

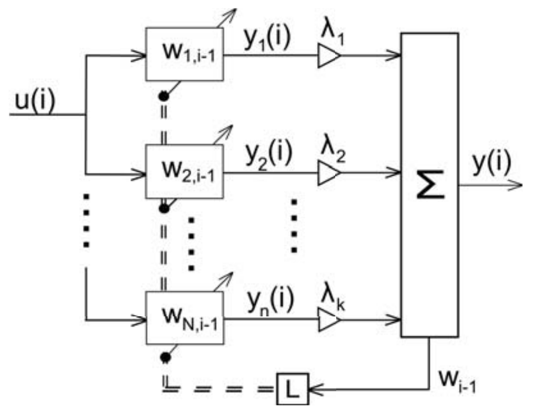
Luiz F. O. Chamon chamon@usp.br
Wilder B. Lopes wilder@usp.br
Cássio G. Lopes cassio@lps.usp.br

ABSTRACT

In combinations of AFs, the component filters usually run independently to be later on combined, leading to a stagnation before reaching the lower error. Conditional transfers of coefficients between the components have been introduced to address this issue. This work proposes a more natural way of accelerating convergence, using cyclic feedbacks of the overall weights to the components instead of unidirectional conditional transfers. It is shown that the cycle length can turn the resulting recursion into an independent combination, a variable step size AF or a hybrid algorithm. Comments on the universality of the approach are presented along with a technique to design the cycle length. Comparisons in different system identification scenarios show the superior performance of the new method.

INTRODUCTION

- Combination of adaptive filters
  - Used when accurate design of a single filter is difficult (e.g., improve the transient/steady-state trade off)
  - *Definition:* set of independent AFs combined by a mixing parameter
  - *Problem:* convergence stagnation
  - *Possible solutions:* Different structures (incremental-cooperative) and conditional transfers of coefficients.



THE STAGNATION PROBLEM

- Adaptive filters

$$w_{n,i} = w_{n,i-1} + \mu_n p_n$$

$w_{n,i} \rightarrow M_n \times 1$ estimate at iteration i of the n^{th} filter;
$p_n = B_n \nabla^* J(w_{n,i-1}) \rightarrow$ update direction dependent on the AF

- Combination of adaptive filters

$$w_{i-1} = \sum_{n=1}^{N} \lambda_n(i) w_{n,i-1}$$

$\lambda_n(i) \rightarrow$ chosen to minimize $\text{E}|e(i)|^2$ subject to $\sum \lambda_n(i) = 1$;
$e(i) = d(i) - u_i w_{i-1} \rightarrow$ overall error;
$u_i \rightarrow 1 \times M$ regressor vector;
$d(i) = u_i w^o + v(i) \rightarrow$ desired signal;
$v(i) \rightarrow$ i.i.d. noise;
$w^o \rightarrow M \times 1$ vector that models the unknown plant.
$e_n(i) = d(i) - u_i w_{n,i-1} \rightarrow$ component filters errors

- Convex combination of LMS filters (CLMS)

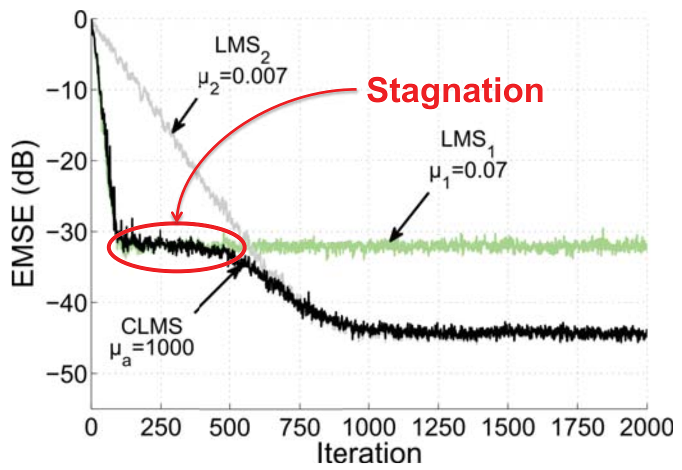
For $N = 2$ and $M_1 = M_2 = M$: e.g., convex combination:

$$w_{i-1} = \lambda(i)\, w_{1,i-1} + (1 - \lambda(i))\, w_{2,i-1}$$

$$\lambda(i) = \frac{1}{1 + e^{-a(i-1)}}$$

$$a(i) = a(i-1) + \mu_a e(i)[y_1(i) - y_2(i)]\lambda(i)[1 - \lambda(i)]$$

$a \in [-a^+, a^+]$



- Transfer of coefficients

$$w_{1,i} = w_{1,i-1} + \mu_1 u_i^* e_1(i)$$

$$w_{2,i} = \alpha[w_{2,i-1} + \mu_2 u_i^* e_2(i)] + (1 - \alpha) w_{1,i-1}$$

$\alpha \rightarrow 1$ and the transfer is conditional (e.g., $\lambda(i) \geq 0.98$)

✓ Effectively addresses the stagnation problem
✗ Requires a conditional test on every iteration
✗ Unidirectional transfer, while in real scenarios the faster AF may change (non-stationary or low SNR environments)

CYCLIC COEFFICIENTS FEEDBACK

$$\lambda(i) = \frac{1}{1 + e^{-a(i-1)}}$$

$$a(i) = a(i-1) + \mu_a e(i)[y_1(i) - y_2(i)]\lambda(i)[1 - \lambda(i)]$$

$$w_{n,i-1} = \delta(i - kL) w_{i-1} + (1 - \delta(i - kL)) w_{n,i-1}$$

$$w_{n,i} = w_{n,i-1} + \mu_n u_i^*(d(i) - u_i w_{n,i-1})$$

$L \rightarrow$ cycle length
$\delta(i) \rightarrow$ the Kronecker delta
$k \in \mathbb{N}$

✓ More natural: provides all filters with the global weights
✓ Feedback is neither directional nor limited to any two filters
✓ Depends uniquely on counters and allow efficient interruption-based implementations

A BRIEF ON ANALYSIS

- The cycle length effect

L: 1 (VSS) ⟶ ∞ (CLMS)

(i) $L \rightarrow \infty \Rightarrow \delta(i - kL) = 0, \forall i \Rightarrow w_{n,i-1} = w_{n,i-1}$

(ii) $L = 1 \Rightarrow w_{n,i-1} = w_{i-1} \Rightarrow$

$$w_i = w_{i-1} + \overline{\mu}(i) u_i^*(d(i) - u_i w_{i-1}),$$

with $\overline{\mu}(i) \triangleq \lambda(i+1)\mu_1 + [1 - \lambda(i+1)]\mu_2$

(iii) $L > 1 \Rightarrow \begin{cases} \text{(ii)}, & i = kL \\ \text{(i)}, & otherwise \end{cases}$

- Comments on universality
  - CLMS is known to be (nearly) universal in steady-state.
  - For $\lambda(i+1) = 0$ the recursion becomes that of the lower misadjustment component. Therefore the VSS may present universality as well.
  - Since (iii) is at each instant equivalent to either (i) or (ii), universality follows.

- Design of the cycle length

Slow adaptation of λ × Use of the overall estimation

In a stationary scenario, the overall weights should be fed back as soon as the faster filter stops converging.

Transient model

From the weigthed variance relation for white Gaussian real-valued inputs, the transient model is

$$\text{E}\|\tilde{w}_{i-1}\|^2 = \gamma^i \text{E}\|w^o\|^2 + \mu\sigma_v^2\sigma_u^2 M \sum_{k=0}^{i-1} \gamma^k$$

$$\gamma = 1 - 2\mu\sigma_u^2 + \mu^2\sigma_u^4(M+2); \quad \tilde{w}_i = w^o - w_i$$

Assuming small step size and high SNR

$$\text{MSE}_{dB}(i) = 10 i \log\gamma + 10 \log[\sigma_d^2 - \sigma_v^2]$$

Steady-state model

$$\text{MSE}(\infty) = \frac{2\sigma_v^2(1 - \mu\sigma_u^2)}{[2 - \mu(M+2)\sigma_u^2]}$$

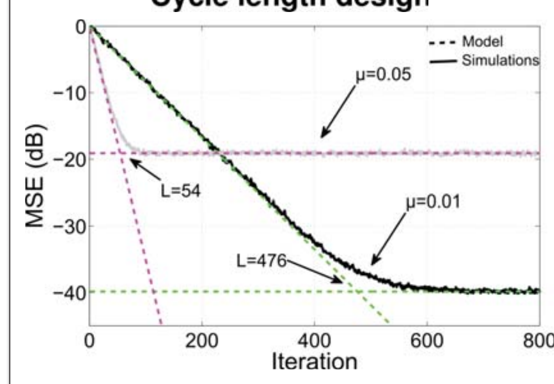
L should be chosen where the model lines cross

SIMULATIONS

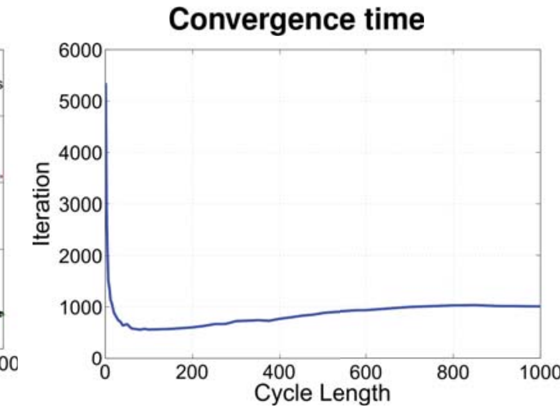
Model validation

Transient model
High SNR/small step size scenario: $\mu = 0.01$, $\sigma_v^2 = 10^{-4}$
Experimental setup scenario: $\mu = \mu_1$, $\sigma_v^2 = 10^{-2}$
Cycle length design
Experimental setup scenario: $\mu = \mu_1$, $\sigma_u^2 = 1$, $M = 7$

Cycle length design

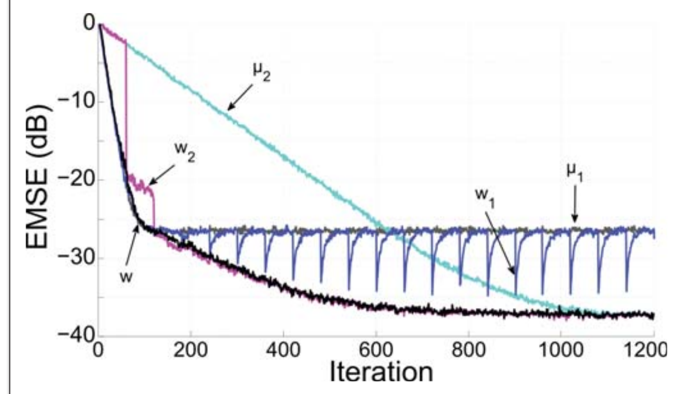


Convergence time

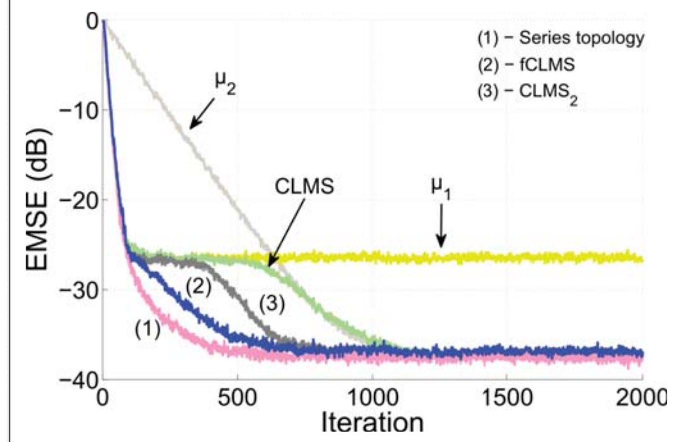


Experimental setup

$\sigma_u^2 = 1$, $\sigma_v^2 = 10^{-2}$ (SNR = 20dB), $\mu_1 = 0.05$, $\mu_2 = 0.005$, $\mu_a^{fCLMS} = 600$, $\mu_a^{CLMS} = 200$, $\mu_a^{CLMS_2} = 100$, $\alpha = 0.9535$, $a^+ = 4$, and $M = 7$

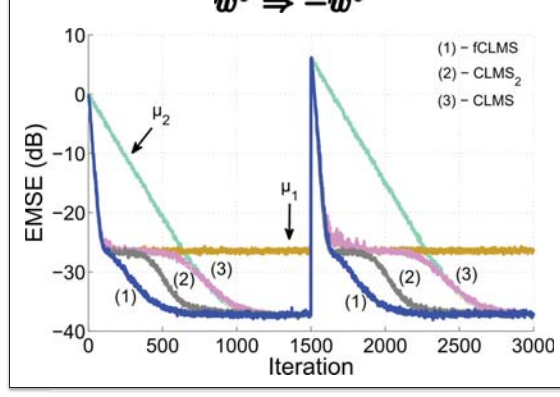


Components behavior

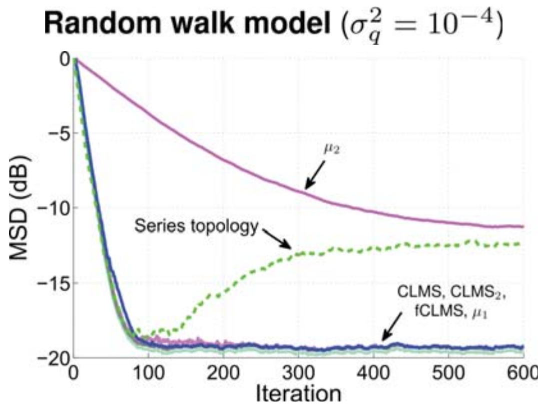


Stationary scenario

$w^o \Rightarrow -w^o$



Random walk model ($\sigma_q^2 = 10^{-4}$)



CONCLUSION

- A novel scheme to overcome the stagnation problem of parallel-independent combinations was proposed: the cyclic coefficients feedback;
- The solution is more natural than conditional transfers of weights;
- For two LMS filters, the structure is equivalent to a CLMS, a VSS algorithm or a hybrid AF, depending on the cycle length;
- A method to design the cycle length was developed and validated;
- Simulations showed that the new algorithm can either match or outperform CLMS, transfer of coefficients and series topology under different scenarios.