THERE'S PLENTY OF ROOM AT THE BOTTOM: INCREMENTAL COMBINATIONS OF SIGN-ERROR LMS FILTERS

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CONTRIBUTIONS

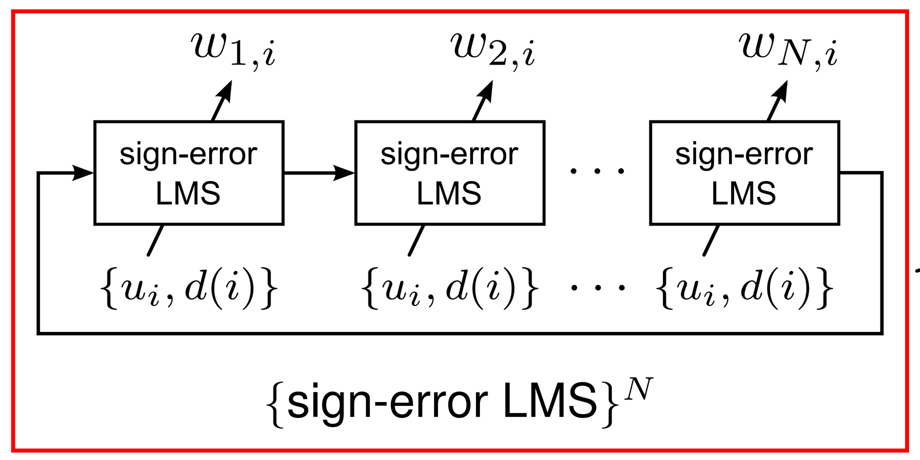
(i) Incremental combination of sign-error LMS filters: {sign-error LMS}^{N}

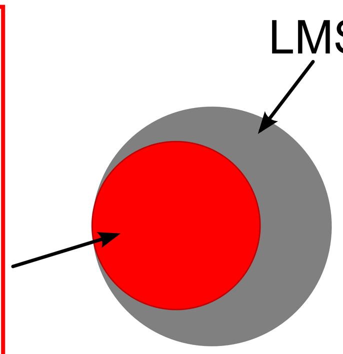
(ii) {sign-error LMS}^{N} $\to$ NLMS, $N \to \infty$

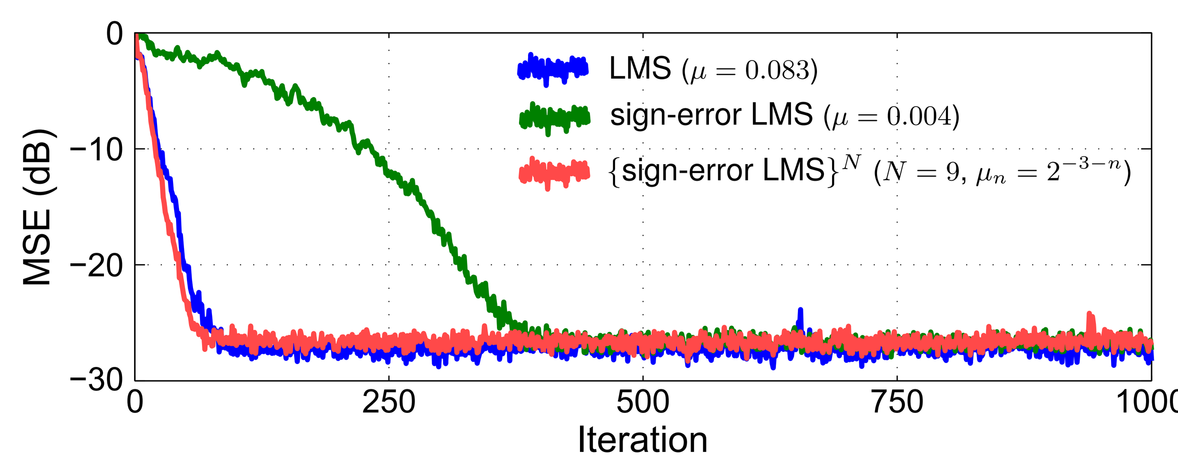
(iii) Design μ_n in {sign-error LMS}^{N} to minimize N

(iv) DR-$\{\{$sign-error LMS$\}^N\}^L$

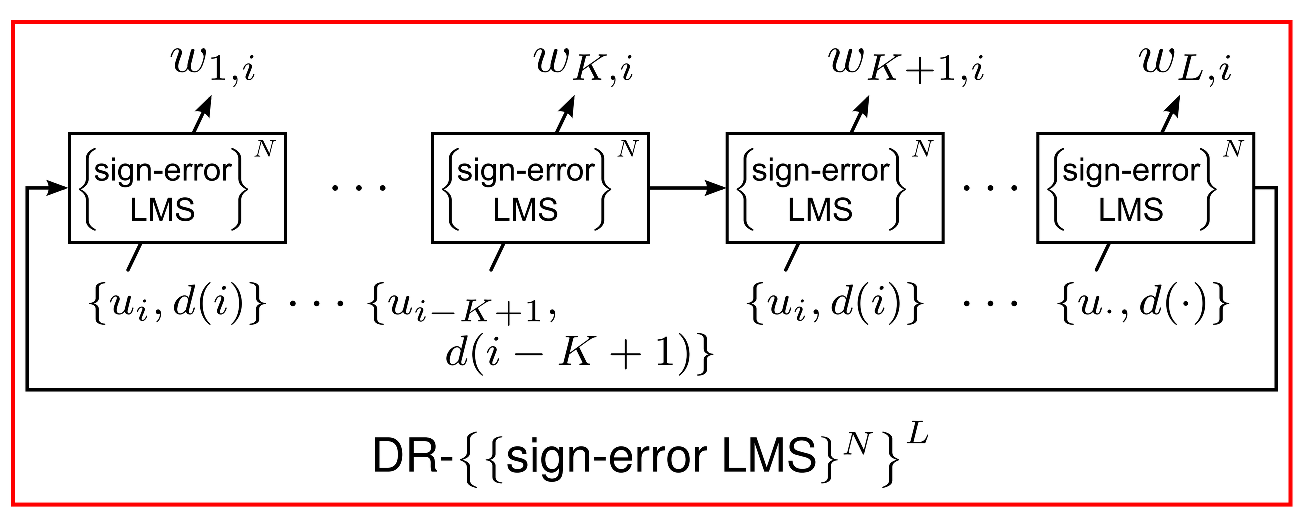
OVERVIEW

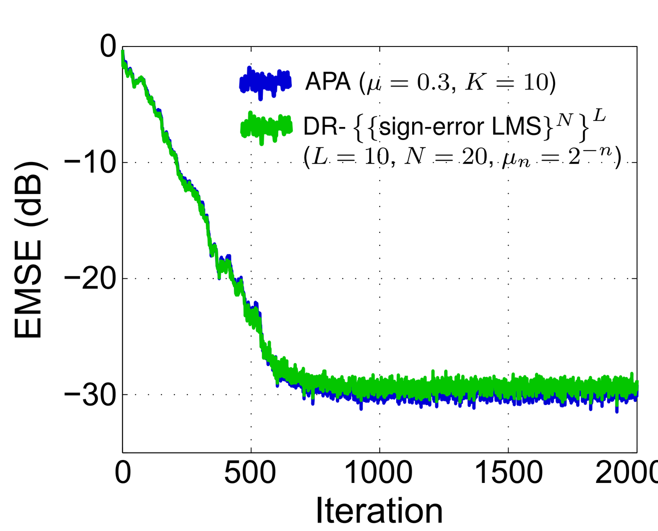




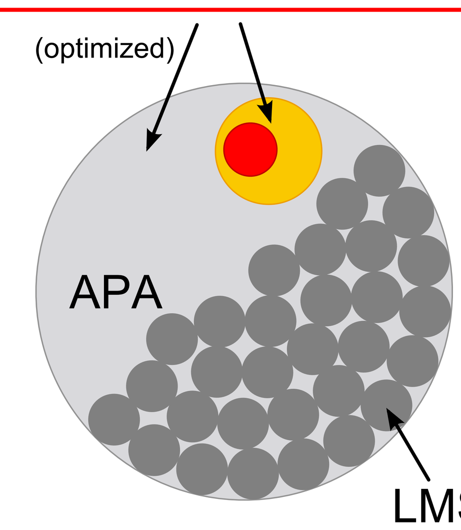


White stationary scenario (fixed point quantization, 16 bits)





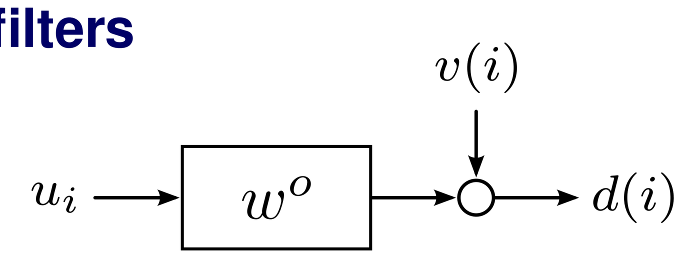
White stationary scenario



Complexity

BACKGROUND

Adaptive filters



$$w_i = w_{i-1} + \mu u_i^T \operatorname{sign}[e(i)]$$
$$w_i = w_{i-1} + \mu u_i^T e(i)$$
$$w_i = w_{i-1} + \frac{\mu}{\|u_i\|^2 + \epsilon} u_i^T e(i)$$
$$w_i = w_{i-1} + \mu U_i^T (U_i U_i^T + \epsilon I)^{-1} e_i$$

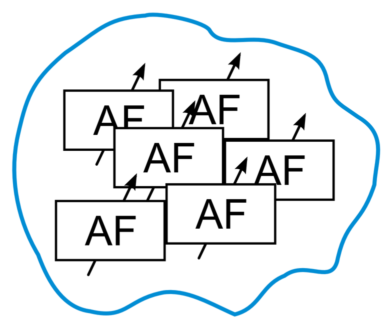
Complexity

$$U_i = \begin{bmatrix} u_i \\ \vdots \\ u_{i-K+1} \end{bmatrix} \quad d_i = \begin{bmatrix} d(i) \\ \vdots \\ d(i-K+1) \end{bmatrix}$$

K, M

Combination of AFs

- **Definition:** set of AFs combined by a supervisor



Better performance

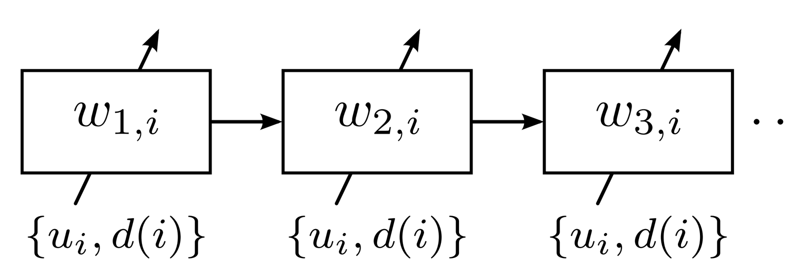
Higher complexity $\left(\approx \sum \mathcal{O}[AF]\right)$

- *"Combination as a complexity reduction technique"*

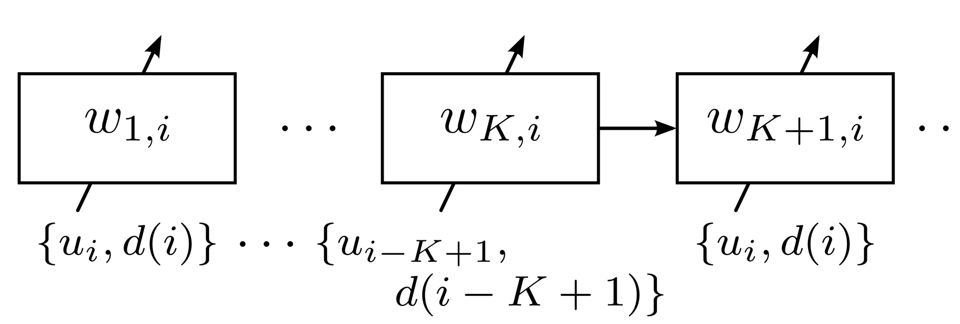
e.g., $\mathcal{O}\left[\text{Combination of LMS}\right] < \mathcal{O}[\text{APA}]$

(same performance)

DR incremental combinations



DATA SHARING



DATA BUFFERING

PLENTY OF ROOM AT THE BOTTOM

Algorithm 1 The {sign-error LMS}^{N}

$\|u_i\|^2 = \|u_{i-1}\|^2 - |u(i-M)|^2 + |u(i)|^2$ ▷ **(1)** $\times$

$y(i) = u_i w_{i-1}$; $e_1(i) = d(i) - y(i)$ ▷ (M) $\times$

$w_{0,i} = w_{i-1}$

for $n = 1, \dots, N$

  $w_{n,i} = w_{n-1,i} + \mu_n u_i^T \operatorname{sign}[e_n(i)]$

  $e_{n+1}(i) = e_n(i) - \mu_n \|u_i\|^2 \operatorname{sign}[e_n(i)]$

end

$w_i = w_{N,i}$

✓ Low complexity: $(M+1)\,\times$ (does not depend on N)

✓ Suited for finite precision & FPGA implementation

{sign-error LMS}^{N} $\to$ NLMS

{sign-error LMS}^{N} $\Rightarrow w_i = w_{i-1} + \bar{\mu}(i) u_i^T \operatorname{sign}[e(i)]$

$$\bar{\mu}(i) = \mu_1 + \sum_{n=2}^{N} \mu_n \prod_{k=1}^{n-1} \operatorname{sign}\left[|e_k(i)| - \mu_k \|u_i\|^2\right]$$

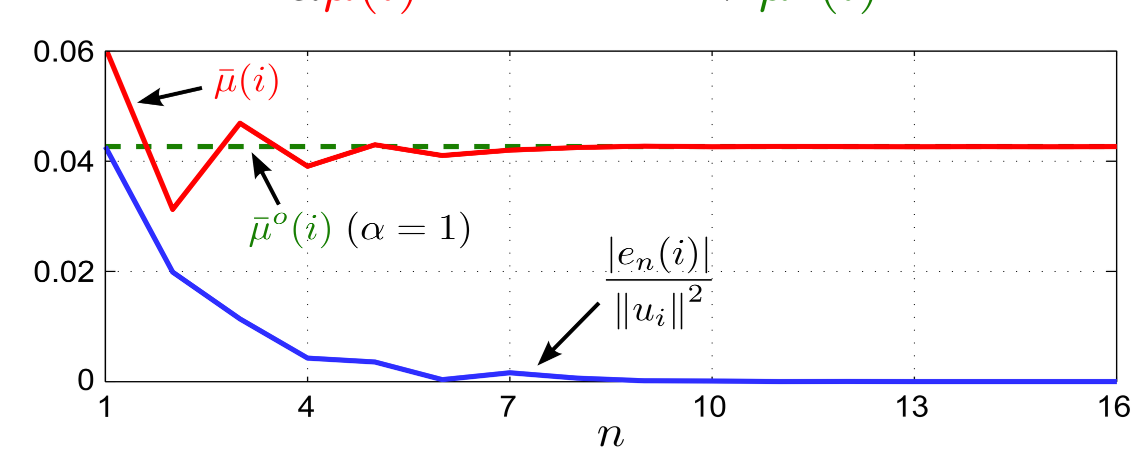
From the sign-error LMS theory:

$$\|w_i\|^2 \leq \|w_{i-1}\|^2 \Leftrightarrow |e(i)| \geq \bar{\mu}(i)\|u_i\|^2$$

$$\Rightarrow \bar{\mu}^o(i) = \alpha \frac{|e(i)|}{\|u_i\|^2}, \quad \alpha \in (0, 1]$$

$$w_i = w_{i-1} + \frac{\alpha}{\|u_i\|^2} u_i^T e(i)$$

$$\alpha \bar{\mu}(i) \xrightarrow{N \to \infty,\ \mu_n \to 0} \bar{\mu}^o(i)$$



Minimizing N: $\mu_n = 2^{-P-n}, \quad P \in \mathbb{Z}$

DR-$\{\{$sign-error LMS$\}^N\}^L$

$$\mathcal{O}\left[\{\text{sign-error LMS}\}^N\right] < \mathcal{O}[\text{LMS}]$$

$$+ \quad \mathcal{O}\left[\text{DR-}\{\text{LMS}\}^L\right] < \mathcal{O}[\text{APA}]$$

$$\mathcal{O}\left[\left\{\{\text{sign-error LMS}\}^N\right\}^L\right] \ll \mathcal{O}[\text{APA}]$$

(same performance)

SIMULATIONS

$x(i) \sim \mathcal{N}(0, 1) \quad v(i) \sim \mathcal{N}(0, 10^{-3})$ (white)

White inputs: $u(i) = x(i)$

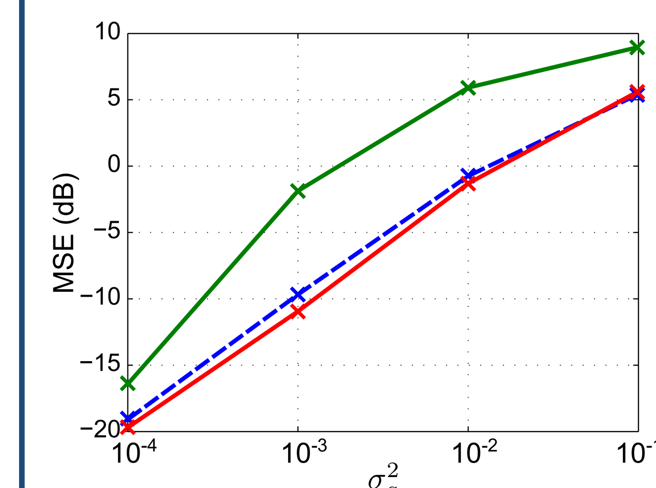
Correlated inputs: $u(i) = \beta u(i-1) + \sqrt{1-\beta^2}\, x(i)$, $\beta = 0.95$

$\mathcal{Q}$: fixed point, 16 bits, $F = 13$ bits (signed Q2.13)

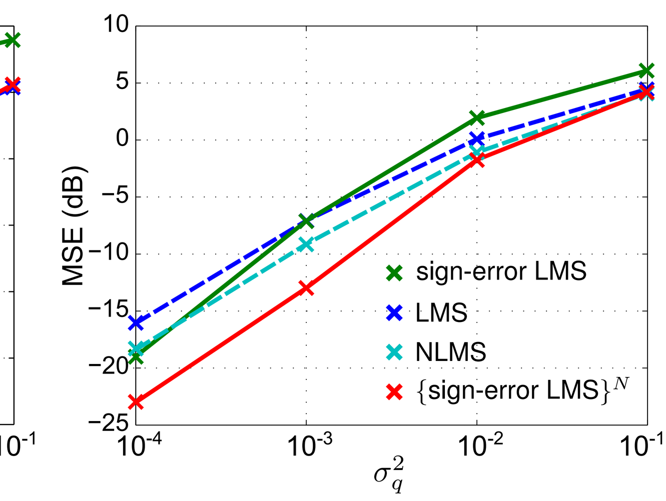
{sign-error LMS}^{N}: nonstationary scenario

($M = 10$, fixed point quantization, 16 bits)

$w_i^o = \mathcal{Q}[w_{i-1}^o + q_i] \quad q_i \sim \mathcal{N}(0, \sigma_q^2 I)$



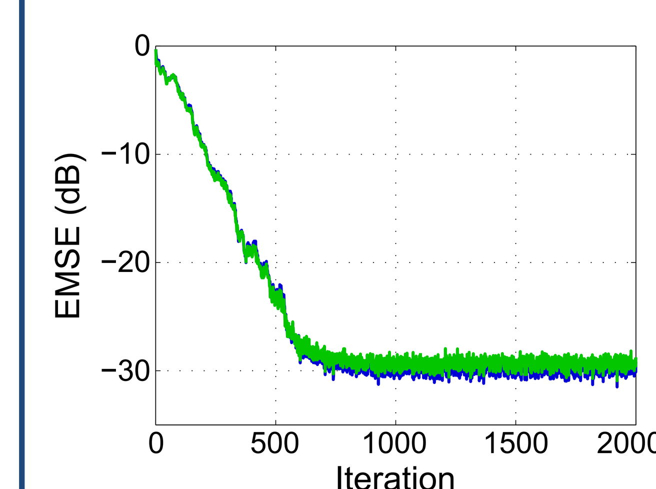
White inputs



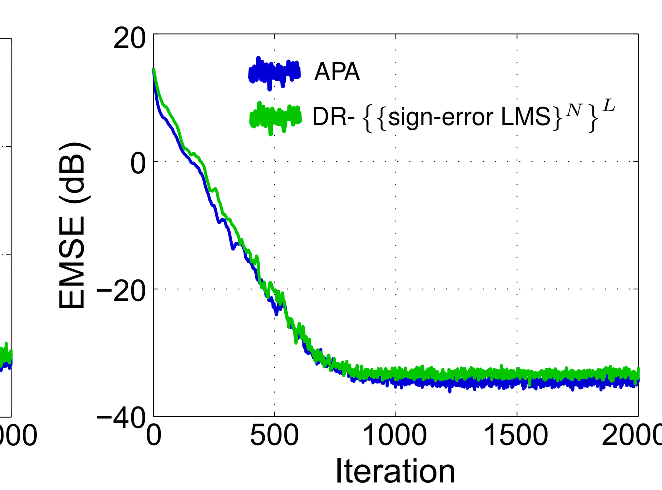
Correlated inputs

$\{\{$sign-error LMS$\}^N\}^L$ and APA

($M = 100$, $K = 10$, double precision)



White inputs



Correlated inputs

AF	$\times$
Standard APA	$(K^2+2K)M + K^3 + K = 13010$
DCD-APA	$M + K^2 + 3K + 2 = 232$
DR-$\{$LMS$\}^L$	$(2M+1)L = 6030$
DR-$\{\{$sign-error LMS$\}^N\}^L$	$2M + K - 1 = 209$

ACKNOWLEDGEMENT

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