18TH INTERNATIONAL CONGRESS ON SOUND & VIBRATION
ICSV18
RIO DE JANEIRO
BRAZIL
10–14 JULY 2011

AN SVD-BASED MIMO EQUALIZER APPLIED TO THE AURALIZATION OF AIRCRAFT NOISE IN A CABIN SIMULATOR

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Noise and vibration are important environmental parameters in an aircraft cabin. However, studying their influence on the comfort of passengers in real flights is difficult and expensive, thus making an aircraft cabin simulator indispensable. Following preliminary work on the development of such a simulator, the problem of reliably reproducing acoustic signals in a simulator is addressed from a system equalization point of view. At first, the transfer-function matrix is assessed using single-loop identification. Then, a filter bank is designed in order to inverse the aforementioned system. The method proposed in this paper applies Singular Value Decomposition (SVD) in the frequency domain as a robust inversion technique for the design of Multiple Input-Multiple Output (MIMO) equalizers. Empirical results show that the method is able to robustly invert the system, and provides good agreement between desired and reproduced signals.

1. Introduction

Airplanes have become a fundamental means of transportation due to their energetic efficiency, speed and safety. The United States Bureau of Transportation Statistics estimates that the number of passengers grew almost 30 times in the last three decades, so that in 2007 136 billions passengers-km were flown [1]. This increase in demand has turned the industry's attention to the comfort of aircraft cabins, to which noise plays a fundamental role [2]. However, the difficulties and costs associated with studying noise comfort in real flights have made essential the development of a simulator capable of reproducing and controlling the aircraft cabin acoustic environment.

The process of rendering audible the sound field of a source in a given space is called *auralization*. This procedure involves the use of mathematical models, measurements and simulations to reproduce accurately the sound from a room in another one, guaranteeing the same auditory sensation to the listener as if being in the original space [3].

This paper describes the development of an equalization technique for the monaural reproduction of aircraft noise in a cabin simulator. First, aircraft noise is analyzed and the reproduction system of the cabin simulator is described. Then, acoustic MIMO equalization is studied in a matrix

This research is part of the "Cabin Comfort" project at the University of São Paulo, Brazil, supported by the São Paulo Research Foundation (FAPESP).

framework and a design solution based on SVD is proposed. Finally, empirical results are provided to validate the equalizers performance.

2. Aircraft noise

The main sources of noise in an airplane are the propulsion system and the aircraft structure. The former generates both narrow band, due to the circular movement of the blades, and wide band, from air flow in the engines intakes, frequency components, whereas the structure-borne noise (aerodynamic noise) is exclusively broad band. Even though cabin noise is dominated by the engines, smaller sources contribute as well, e.g. air conditioning and pressurization systems [4].

The sound pressure level (SPL) inside an aircraft heavily depends on flight characteristics (altitude, speed...), seat position and the aircraft itself. Modern airplanes usually have levels below 80 dBA, 70 dBA being considered a comfortable value. Furthermore, the front of the aircraft tends to be quieter than the back, where structural and engine contributions increase [5].

Aircraft cabin noise is mainly composed of low frequencies (5 to 200 Hz), showing a slow decay beyond 500 Hz. Narrow peaks due to the propulsion system are present around 100 Hz, and smaller peaks in high frequencies can usually be attributed to the air conditioning system.

3. Cabin simulator

The simulator was built at the University of São Paulo, Brazil, around a cabin mock-up composed of twenty economic class seats from a commercial passenger aircraft (Fig. 1). In addition to noise reproduction, the simulator is equipped with an air conditioning and vibration system capable of providing the user with a fully immersive experience.

The acoustic reproduction system is responsible for mimicking the noise environment of an aircraft cabin based on recordings made during real flights. It is composed of four 12” loudspeakers (woofers), installed in the overhead compartments (Fig. 2a), and ten 6x9” loudspeakers (mid-high), installed under every pair of seats (Fig. 2b). Each transducer is driven independently, allowing full control over their individual contributions to the sound field. The monitoring of the reproduced noise is based on ten omnidirectional microphones placed between each pair of seats (Fig. 2c).

4. Singular Value Decomposition and the pseudo-inverse

Singular value decomposition is a matrix factorization defined for any $A \in \mathbb{C}^{MxN}$ as

$$A = U\Sigma V^*, \quad (1)$$

where $U \in \mathbb{C}^{MxM}$ and $V \in \mathbb{C}^{NxN}$ are the unitary matrices of the eigenvectors of AA^* and A^*A, respectively, $\Sigma = diag\{\sigma_i(A)\} \in \mathbb{R}^{MxN}$ captures in descending order the square root of the eigenvalues of AA^*, called singular values, and $(\cdot)^*$ denotes the conjugate transpose [6]. Note that, contrary to eigenvalue decomposition, SVD exists for every matrix, even if not square.

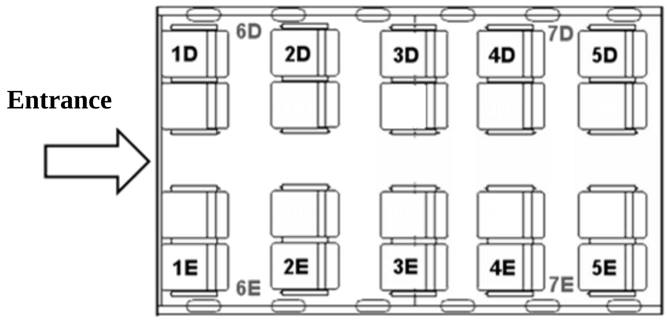

Figure 1. Diagram of the aircraft cabin simulator.

(a) (b) (c)

Figure 2. Reproduction system: (a) Low range speaker; (b) Mid-high range speaker; (c) Measurement microphone

Among its numerous applications, SVD provides the solution to the rank approximation problem defined as finding $B \in \mathbb{C}^{MxN}$ of rank $r < rank(A)$ which minimizes $\|A - B\|_F$, where $rank(\cdot)$ is the size of the largest non-vanishing minor and $\|A\|_F = \sqrt{\mathrm{Tr}(AA^*)}$ is the Frobenius norm [6]. The Eckart-Young theorem [7] shows that the truncated SVD $A_r = U\Sigma_r V^*$, with $\Sigma_r = diag\{\sigma_i(A), 0, \ldots, 0\}$, $i \leq r$, is the optimal solution for this problem with

$$\min_{B|rank(B)=r} \|A - B\|_F = \|A - A_r\|_F = \sigma_{r+1}(A). \tag{2}$$

Equation (2) leads to another application of SVD for robustly inverting matrices using a concept known as the pseudoinverse. When the inverse of a matrix exists, it can be evaluated through $A^{-1} = V\Sigma^{-1}U^*$. However, if any of the singular values is null or close to the precision of the system, this inverse cannot be computed. The pseudoinverse solves this problem inverting a lower rank approximation of the original matrix, namely

$$A^+ = V\Sigma^+U^*, \tag{3}$$

where A^+ denotes the pseudoinverse and $\Sigma^+ = diag\{\sigma_1^{-1}(A), \ldots \sigma_n^{-1}(A), 0, \ldots, 0\}$, so that $\sigma_{n+1}(A) < k < \sigma_n(A)$, in other words, all singular values smaller than k are zeroed instead of inverted. Doing so leaves out directions highly corrupted by round off errors, allowing for a much more robust solution to the inversion problem. Moreover, SVD pseudoinverse is a solution to the total least square problem [8].

5. Acoustic MIMO equalization

The equivalent MIMO system of the simulator is represented in Fig. 3, where h_{mn} is the transfer-function (TF) from speaker n to microphone m, τ_m are arbitrary delays, e_m are error signals, g_n are equalizing filters and s is the desired signal. Due to the linear nature of sound propagation [9], this model can be summarized by a matrix equation in terms of the Fourier representation of each component, namely

$$\mathcal{M}(j\omega) = H(j\omega)diag\{G(j\omega)\}S(j\omega)col\{1\}, \tag{4}$$

where $\mathcal{M}(j\omega)$ is the $M\ x\ 1$ complex vector that captures the frequency response of the microphone signals, $H(j\omega) = \mathcal{F}[h_{mn}]$ represents the $M\ x\ N$ transfer-function matrix (TFM) at frequency ω, $G(j\omega)$ is the $N\ x\ 1$ vector of frequency response for the equalizers, $S(j\omega)$ is the Fourier transform of the desired signal and $col\{\cdot\}$ is a column vector. Furthermore,

$$E(j\omega) = \mathcal{M}(j\omega) - S(j\omega)T(j\omega), \tag{5}$$

with $E(j\omega)$ and $T(j\omega) = [e^{-j\omega\tau_1} \quad \ldots \quad e^{-j\omega\tau_M}]^T$ being the $M\ x\ 1$ complex vectors of errors and delays, respectively. The dependency on $j\omega$ will be implied from now on to simplify the deductions.

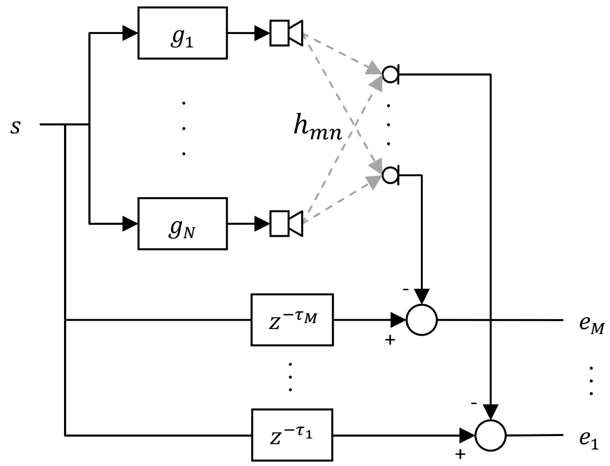

Figure 3. Diagram of the aircraft cabin simulator.

The error defined in (5) will reach its minimum value when the signals at the microphones are, except for a delay, identical to the desired signal, i.e. $\mathcal{M}^o = ST$. Note that no synchronization is imposed, i.e. $\tau_1, \ldots, \tau_M$ may be different, though this constraint could easily be included in the proposed solution. From (4), $\mathcal{M}^o$ can only be obtained if

$$HG = T \Leftrightarrow \begin{cases} |HG| = col\{1\} \\ \phi(HG) = col\{-\omega\tau_m\} \end{cases}, \tag{6}$$

where $|\cdot|$ is the modulus and $\phi(\cdot)$, the phase of a complex number.

Equation (6) has the form of a complex linear system, which assumes different solutions depending on the form of the coefficient matrix H [6]:

i. If $M < N$, the system is underdetermined and has infinite solutions. Usually, minimum-norm constraints are imposed to the missing degrees of freedom and the solution becomes $G^o = H^*(HH^*)^{-1}T$.
ii. If $M = N$, the system is determined and has a unique solution given by $G^o = H^{-1}T$.
iii. If $M > N$, the system is overdetermined and a unique solution can be found through least squares, namely $G^o = (H^*H)^{-1}H^*T$.

Even though (ii) allows the best possible equalization of the system, it is very sensitive to noise in the TF measurements, so that it is usually better to adopt a least squares approach (iii) [10]. In all the above cases, G^o is optimal in least square sense.

The solutions presented to the linear system (6) require somehow for the matrix H to be inverted, so that it cannot be singular or, from a numerical point of view, ill-conditioned [6]. However, proximity between transducers or sensors [11], symmetry and reverberation [12] tend to augment the correlation between elements of the TFM, thus worsening its conditioning.

A well known method for addressing this issue is called regularization, introduced in the acoustic framework by [13]. Constraining the speakers output power, a mixed cost function $J = E^*E + \beta G^*G|S|^2$, where β is the regularization factor, is minimized, changing, for example, (iii) into

$$G_R = (H^*H + \beta I)^{-1}H^*T. \tag{7}$$

From a purely algebraic point of view, this approach is equivalent to summing β to each eigenvalue of H^*H, guaranteeing that the matrix is not singular and ameliorating its conditioning [6]. However, it is important that the regularized matrix does not differ much of the original one, otherwise degrading the solution of (6). Therefore, a compromise exists in the choice of β, that must remain small (usually $\beta \ll 1$) to ensure the equalization performance of the inverse while being large enough to significantly improve the matrix conditioning.

A numerical evaluation of the conditioning of a matrix is provided by the condition number, defined as $\kappa_2(A) = \sigma_{max}/\sigma_{min}$. As κ_2 grows, the matrix becomes closer to singular in finite precision and the less robust the inversion will be [8]. From (7),

$$\kappa_2(H^*H + \beta I) = \frac{\sigma_{max}^2+\beta}{\sigma_{min}^2+\beta} \approx [\kappa_2(H)]^2 - \frac{(\sigma_{max}^2-\sigma_{min}^2)}{\sigma_{min}^4}\beta. \quad (8)$$

which shows that β actually influences the squared condition number. This is due to (7) requiring the computation of H^*H, which has $\kappa_2(H^*H) = [\kappa_2(H)]^2$, hence deteriorating the conditioning and leading to $rank(H^*H) < rank(H)$ if any $\sigma_i(H)$ is close to the machine precision [8]. Note that the same problem arises in the underdetermined case (i).

5.1 The decoupling equalizers

As a mean to avoid the aforementioned issues, a solution based on the SVD pseudoinverse is proposed to the acoustic MIMO equalization problem. Defining $H(j\omega) = U(j\omega)\Sigma(j\omega)V^*(j\omega)$, the SVD of H at frequency ω, and using (3), the solution to (6), independent of the matrix form, yields

$$G_{SVD} = H^+T = V\Sigma^+U^*T. \quad (9)$$

It can be shown that if the TFM is full rank, i.e. if its inverse exists, (9) provides a numerically robust way to compute G^o in (i) and (iii), since the iterative SVD algorithm does not compute H^*H explicitly [8]. Furthermore, truncated SVD is guaranteed by (2) to provide the best low rank version of the original matrix, while decreasing the condition number as in

$$\kappa_2(H_k) = \frac{\sigma_{max}(H_k)}{\sigma_{min}(H_k)} < \frac{\sigma_{max}}{k} = \kappa_2(H) - \frac{\sigma_{max}(k-\sigma_{min})}{\sigma_{min}k}. \quad (10)$$

The advantages presented in this section come at the higher computational cost of SVD, when compared to the simplicity of regularization, one of the main reasons behind its popularity [6]. Additionally, both methods suffer from the fact that they rely on the coordination of the actuators in order to reproduce the signals. It means that the loss of one single speaker can affect the reproduction over the whole system.

6. Results

The TFM of the system was evaluated using single-loop identification, i.e. individually exciting the speakers while measuring their response on each microphone. Using discrete Fourier transform, the frequency responses were obtained (e.g. Fig. 4) and organized in $H(j\omega)$. The test signal was an exponential sweep of 30 seconds from 10 Hz to 22 kHz [14].

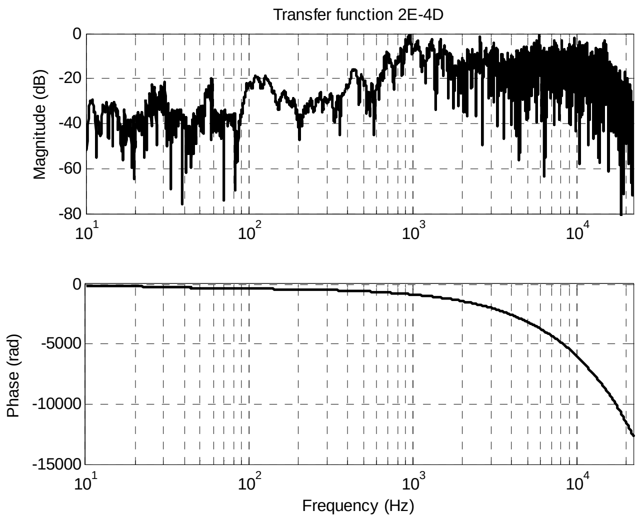

Figure 4. Transfer-function from the speaker in seat 2E to the microphone in 4D.

Using (9), equalizing filters were designed and tested in the mock-up presented in Section 3. The results for different singular value thresholds k are presented in Fig. 5, along with the target third-octave spectrum. The scheme has shown good agreement in mid to low bands, loosing quality in higher and lower frequencies. This effect is due to the difficulty in reproducing these ranges in the simulator, easily observed by the magnitude drops in Fig. 4. Improvements can be made reducing the value of k, allowing for a more complete inversion of the original matrix, thus improving the equalization.

Although not taken directly into account during the equalizers design, the central aisle is also an important listening point to any passenger standing up. Therefore, measurements were taken in the mock-up corridor and compared to the reference points and the target signal for $k = 0{,}05$. Fig. 6 shows almost no difference between the central aisle and the seats, even though the reproduction at those points was not part of the design constraints.

Lastly, a study of the variation between seats of the equivalent sound level is presented in Fig. 7. In this context, a higher k clearly provides less disparity and a better agreement with the target level. Since increasing the singular value threshold keeps only directions that are easier to reproduce, parts of the TFM that were highly corrupted by noise are left out instead of inverted, explaining the uniformity of the sound levels. Additionally, the truncated SVD still accounts for most of the correlation speaker-microphone, therefore maintaining the consistency with the original sound level.

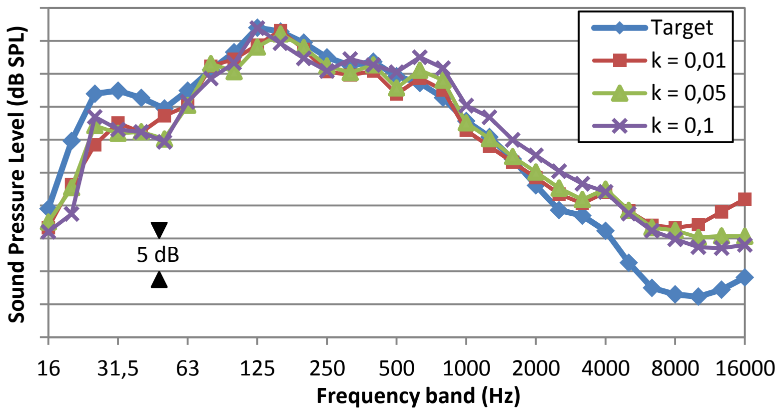

Figure 5. Average spectrum at the seats for different $\boldsymbol{k}$ in third-octave bands.

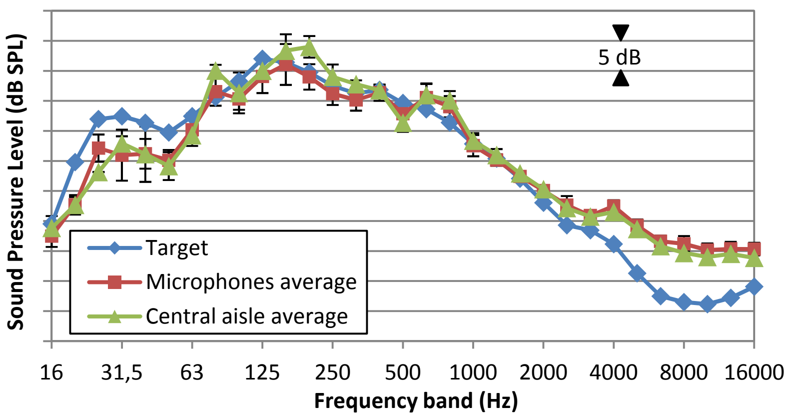

Figure 6. Comparison between spectrum at the seats and in the central aisle.

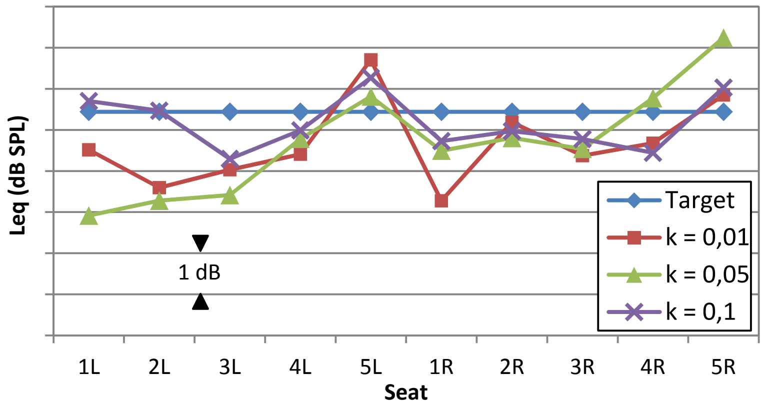

Figure 7. Equivalent sound levels at each seat for different $\boldsymbol{k}$.

7. Conclusions

The auralization of aircraft noise in a cabin simulator was studied under a MIMO equalization perspective. Based on the optimal characteristics of the factorization, an SVD-based pseudoinverse method was proposed for the design of an equalizers bank. Preliminary empirical results proved the solution to be robust in finite precision, providing good agreement between desired and reproduced signals in a wide range of frequencies. In the low and high end of the third-octave spectrum, the mismatch increases due to the difficulty of the system to reproduce these frequencies. In order to improve this, future studies will include the use of relative thresholds (fixed condition number), saturation and adaptive singular values for nonstationary systems.

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