FIR–IIR adaptive filters hybrid combination

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To enhance the performance in IIR system identification scenarios, a hybrid combination of FIR and IIR adaptive filters (AFs) via a supervisor that senses which one is performing best is proposed. The FIR-LMS AF is short, providing fast and robust convergence, whereas the IIR-LMS AF is slow but accurate. The stagnation effect caused by the different convergence rates is tackled through cyclic weight transfers FIR → IIR, which also ensure good tracking properties. A design technique for the transfers cycle length is proposed, providing good convergence while keeping computational cost low.

Introduction: In this Letter, we introduce an FIR–IIR AFs (adaptive filters) combination in the system identification setup which shows improved performance over an ordinary IIR AF while keeping the computational complexity low by skipping stability checks. To achieve this goal, the FIR AF (or 'guide') is designed to be fast and robust while the IIR AF has a small step size, thus it is accurate. In addition, a cyclic transfer of weights based on the balanced order reduction [1] is performed. The resulting system converges as fast as the FIR guide while achieving the lowest excess mean squared error (EMSE) of the IIR AF, and can be extended to other applications, as an adaptive prediction [2].

FIR and IIR adaptive filtering: Consider the system identification scenario in Fig. 1, in which an AF $H_i(z)$ tries to identify a plant $H^o(z)$, both IIR with the same order M. The desired signal $d(i)$ is given by [3]

$$d(i) = y^o(i) + v(i) = \boldsymbol{x}_i^o \boldsymbol{w}^o + v(i) \tag{1}$$

with the unknown weights $\boldsymbol{w}^o$ and the 'plant regressor' $\boldsymbol{x}_i^o$ vectors set to

$$\begin{aligned} \boldsymbol{w}^o &= \left[-a_1^o \;\; -a_2^o \;\; \ldots \;\; -a_{M-1}^o \;\; -a_M^o \;\; b_0^o \;\; b_1^o \;\; \ldots \;\; b_{M-1}^o \;\; b_M^o\right]^{\mathrm{T}} \\ \boldsymbol{x}_i^o &= \left[y^o(i-1)\; y^o(i-2)\; \ldots \; y^o(i-M)\; u(i)\; u(i-1)\; \ldots \; u(i-M)\right] \end{aligned}$$

where $u(i)$ is a Gaussian input signal, $\{b_k^o\}$ and $\{a_k^o\}$ are the unknown feedforward and feedback coefficients, $v(i)$ is a Gaussian zero-mean white noise. Similarly, define the AF weights and regressor vectors $\boldsymbol{w}_i$ and $\boldsymbol{x}_i$ as

$$\boldsymbol{w}_i = \left[-a_1(i) \;\; -a_2(i) \;\; \ldots \;\; -a_M(i)\; b_0(i)\; b_1(i)\; \ldots \; b_M(i)\right]^{\mathrm{T}}$$

and

$$\boldsymbol{x}_i = [y(i-1)\; y(i-2)\; \ldots \; y(i-M)\; u(i)\; \ldots \; u(i-M)]$$

so that the AF output is $y(i) = \boldsymbol{x}_i \boldsymbol{w}_{i-1}$ [3, 4]. In the output error LMS (IIR-LMS) algorithm [3, 5], the squared error $e^2(i) = (d(i) - y(i))^2$ is minimised via the stochastic gradient $\nabla_w e^2(i) = -\boldsymbol{\varphi}_i^{\mathrm{T}} e(i)$, where the 'filtered regressor' $\boldsymbol{\varphi}_i$ is defined as $\boldsymbol{\varphi}_i = \boldsymbol{x}_i - \sum_{k=1}^{M} a_k(i)\boldsymbol{\varphi}_{i-k}$ [5] and then the IIR-LMS update rule arises as [3, 5]

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu \boldsymbol{\varphi}_i^{\mathrm{T}} e(i) \tag{2}$$

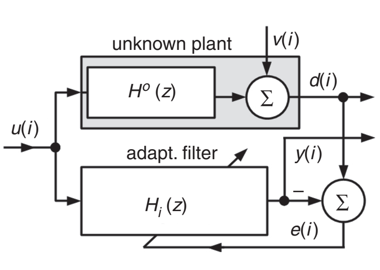

Fig. 1 *System identification*

IIR-LMS may be employed, but its normalised version, the IIR-NLMS algorithm, is usually preferred as it is less sensitive to underdamped or clustered poles in $H^o(z)$ [6]

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu \frac{\boldsymbol{\varphi}_i^{\mathrm{T}}}{\|\boldsymbol{\varphi}_i\|^2 + \epsilon} e(i) \tag{3}$$

in which ϵ is a small regularisation factor. With $\{a_k = 0\}$, the standard FIR-LMS is retrieved from the IIR-LMS aforementioned

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu \boldsymbol{u}_i^{\mathrm{T}} e(i) \tag{4}$$

where $\boldsymbol{u}_i = [u(i)\; u(i-1) \ldots u(i-M_1+1)]$ is $M_1 \times 1$ in the FIR case.

Both update rules of (3) and (4) provide unbiased estimates at a low computational cost, but the former may require stability checks to prevent unstable poles in $H_i(z)$ [5, 7, 8], making the adaptation computationally costly. However, in this work the IIR AF is implemented with a small step-size to provide accuracy in the steady-state, which leads to an exponential stability [9] and, therefore, makes the checks unnecessary. In addition, stability is further enforced via AF combinations, as explored in the sequel.

Combinations of adaptive filters: This Letter proposes a hybrid FIR–IIR combination ('FIIR cell') as in Fig. 2, in which a fast and robust FIR-LMS (AF1) is combined with an accurate but slow IIR-NLMS (AF2), generating the overall filter output $y(i) = \lambda(i)y_1(i) + [1-\lambda(i)]$ $y_2(i)$, where $y_k(i)$ is the kth AF output, $\lambda(i) = 1/(1+\mathrm{e}^{-a(i-1)})$ is the convex supervisor and $a(i)$ is adapted via a gradient-descent rule [10, 11]

$$a(i) = a(i-1) + \mu_a e(i)\big[y_1(i) - y_2(i)\big]\lambda(i)\big[1-\lambda(i)\big] \tag{5}$$

with $e(i) = d(i)–y(i)$ the combination global error and μ_a a step size.

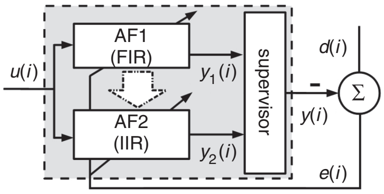

Fig. 2 *Filters combination*

Such a scheme drives the global filter $y(i)$ to the convergence rate of the fast AF and the steady-state error of the accurate filter, although a stagnation effect takes place until the latter outperforms the former one [11]. When both AFs are FIR, this can be mitigated via direct $\mathrm{FIR}_1 \to \mathrm{FIR}_2$ weights transfer. When combining FIR with IIR filters, the stagnation effect may be much worst, and weights transfers are not trivial. Transfers IIR → FIR are simple, but not wise, so that the robustness is enforced. On the other hand, transfers FIR → IIR are mathematically involved, but may considerably enhance overall stability and convergence. We address this issue by introducing a projection function $\mathcal{P}$:FIR → IIR in the context of AF combinations that maps AF1 into AF2 whenever convenient. There are many ways to define $\mathcal{P}$; here, we resort to 'balance order reduction' [1], implemented via the efficient eigen-decomposition of the FIR AF weights Hankel matrix [12], at complexity $O((M_1+1)^2\log(M_1+1))$, with M_1 small (FIR-AF). The transfers AF1 → AF2 are performed cyclically, every L iterations, whenever AF1 is better than AF2, as tracked by $\lambda(i)$. Herewith, complexity per cycle is further reduced and non-stationaries are addressed, particularly abrupt changes in $\boldsymbol{w}^o$. The resulting algorithm is then given by

$$\boldsymbol{w}_{1,i} = \boldsymbol{w}_{1,i-1} + \mu_1 \boldsymbol{u}_i^{\mathrm{T}} e_1(i) \tag{6}$$

$$\boldsymbol{w}_{2,i-1} = \begin{cases} \delta_L \mathcal{P}\big(\boldsymbol{w}_{1,i-1}\big) + (1-\delta_L)\boldsymbol{w}_{2,i-1}, & \lambda(i) \geq \beta \\ \boldsymbol{w}_{2,i-1}, & \lambda(i) < \beta \end{cases} \tag{7}$$

$$\boldsymbol{w}_{2,i} = \boldsymbol{w}_{2,i-1} + \mu_2 \frac{\boldsymbol{\varphi}_i^{\mathrm{T}}}{\|\boldsymbol{\varphi}_i\|^2 + \epsilon} e_2(i) \tag{8}$$

$$a(i) = a(i-1) + \mu_a e(i)\big(y_1(i) - y_2(i)\big)\lambda(i)\big(1-\lambda(i)\big) \tag{9}$$

in which $\delta_L = \delta(r)$ is the Kronecker delta with $r = i \bmod L$, $0 \ll \beta < 1$ is a constant that determines when AF1 is better than AF2, $e_k(i) = d(i) - y_k(i)$, $y_1(i) = \boldsymbol{u}_i \boldsymbol{w}_{1,i-1}$, $y_2(i) = \boldsymbol{x}_i \boldsymbol{w}_{2,i-1}$.

Cycle length L: We explore the energy conservation relation (ECR) [4] to devise a simple yet efficient design technique for L. First, the FIR-LMS learning is approximated by a straight line (in dB) that captures the AF initial convergence rate (see Fig. 3*a*). When this curve reaches the noise floor $10\log\sigma_v^2$ at $i = L$, the AF1 steady state is declared and the transfer $\mathcal{P}$:$\boldsymbol{w}_{1,i-1} \to \boldsymbol{w}_{2,i-1}$ is performed, if AF1 is better than AF2.

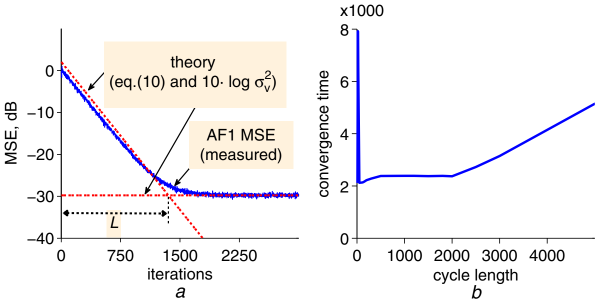

Fig. 3 *Cycle length design*

a The design principle
b Convergence time over

Applying the ECR over the standalone FIR-LMS filter (AF1) leads to the weighted variance relation [4], which provides a model for the AF1 transient evolution, assuming a real valued Gaussian white input $u(i)$; i.e.

$$E\|\tilde{\boldsymbol{w}}_{1,i-1}\|^2 = \gamma^i E\,\|\boldsymbol{w}_1^o\|^2 + \mu_1\sigma_v^2\sigma_u^2(M_1+1)\sum_{k=0}^{i-1}\gamma^k \qquad (10)$$

where $\gamma \triangleq 1 - 2\mu\sigma_u^2 + \mu_1^2\sigma_u^4(M_1+4)$, $\boldsymbol{w}_1^o$ is the FIR truncated from the IIR $H^o(z)$, $\tilde{\boldsymbol{w}}_{1,i} = \boldsymbol{w}_1^o - \boldsymbol{w}_{1,i}$. Equation (10) can be approximated by $E\,\|\tilde{\boldsymbol{w}}_{1,i-1}\|^2 \simeq \gamma^i E\,\|\boldsymbol{w}_1^o\|^2$. As $e_1(i) = \boldsymbol{u}_i\,\tilde{\boldsymbol{w}}_{1,i-1} + v(i)$, the MSE $Ee_1^2(i)$ equals $\sigma_u^2 E\,\|\tilde{\boldsymbol{w}}_{1,i-1}\|^2 + \sigma_v^2$. The quantity $\|\boldsymbol{w}_1^o\|^2$ can be estimated from $\|\boldsymbol{w}_1^o\|^2 = \left(\sigma_d^2 - \sigma_v^2\right)/\sigma_u^2$ [4], such that $Ee_1^2(i) \simeq \gamma^i\left(\sigma_d^2 - \sigma_v^2\right)$ and the AF1 approximate transient model becomes (in dB)

$$\mathrm{MSE}_{1,\,\mathrm{dB}}(i) \cong 10i\,\log\,\gamma + 10\,\log\left(\sigma_d^2 - \sigma_v^2\right) \qquad (11)$$

A good estimate for L is then $L = \left\{i|\mathrm{MSE}_1(i) = 10\log\sigma_v^2\right\}$ (see Fig. 3*a*). This simple design is quite efficient, and robust as L varies over a wide range, as Fig. 3*b* depicts. Therein, the convergence time of an FIIR cell is depicted as a function of L, and it is defined as the time required for the combination to reach 85% of the AF2 steady-state error.

Simulation results: In the simulations below, the FIIR cell identifies the plants, the frequency responses of which are in Figs. 4*a* and *b*. Both are particularly hard due to their peculiar poles-zeros setup [6, 13]: the first is a Butterworth system with clustered poles and the second is a notch with underdamped poles. In both cases, $M = 6$, $u(i)$ is a zero mean Gaussian white signal with power $\sigma_u^2 = 1$ and $v(i)$ has a power of $\sigma_v^2 = 1 \times 10^{-3}$.

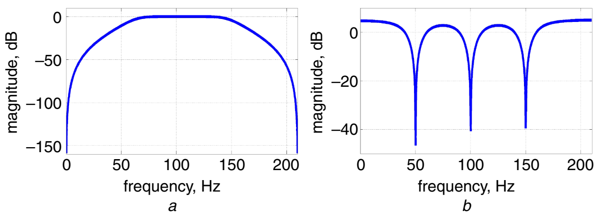

Fig. 4 *Frequency response of simulation scenarios*

a Butterworth scenario
b Notch scenario

The plots that follow show the EMSE of AF1 ('guide'), the FIIR cell with weight transfers ('WT', see (7)) and without ('no-WT'), where the superiority of the former is clear. For reference, the EMSE of a standalone IIR AF is also shown.

In Fig. 5, the combination identifies the Butterworth system with $M_1 = 10$, $\mu_1 = 0.02$ and $\mu_2 = 0.005$. In this case, the error surface of a regular IIR AF exhibits near constant error regions around the global minimum on which gradient-based algorithms adapt slowly [6]. The combination considerably improves the performance, as AF1 guides AF2 towards the vicinity of the minimum rapidly. Only the first iterations are shown as the standalone IIR takes too long to reach the combination's steady-state.

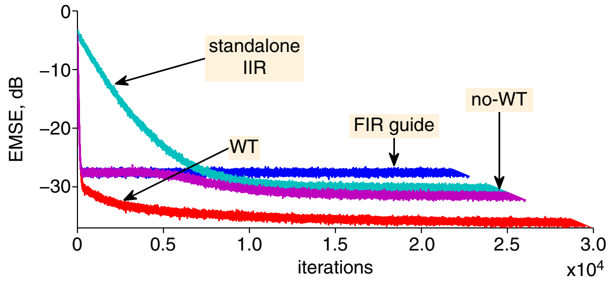

Fig. 5 *Stationary scenario*

In Fig. 6, the combination identifies the notch with $M_1 = 15$, $\mu_1 = 0.007$ and $\mu_2 = 0.03$. When $i = 4 \times 10^4$, the notch frequency changes suddenly (abrupt change in $\boldsymbol{w}^o$). Although the standard IIR nearly stagnated, the FIIR cell responded fast, with a much superior performance.

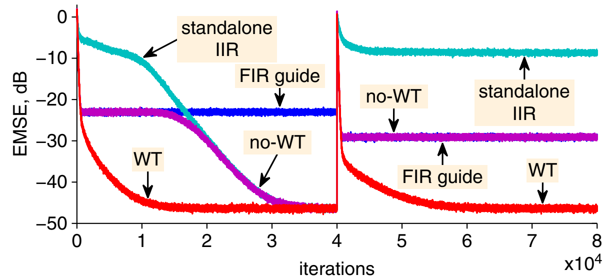

Fig. 6 *Non-stationary scenario*

Conclusion: The FIIR cells are robust, have a superior convergence rate and handle abrupt non-stationarities better than a standard IIR AF while avoiding the stability checks. In stationary scenarios, a properly designed L minimises the number of transfers needed to accelerate AF2 and decreases the overall complexity of the mapping $\mathcal{P}$. The current work involves fast Fourier transform (FFT)-based maps to efficiently replace the function $\mathcal{P}$ employed here.

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One or more of the Figures in this Letter are available in colour online.

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References

1 Beliczynski, B., Kale, I., and Cain, G.D.: 'Approximation of FIR by IIR digital filters: an algorithm based on balanced model reduction', *IEEE Trans. Signal Process.*, 1992, **40**, (3), pp. 532–542
2 Lopes, C.G., Satorius, E.H., Estabrook, P., and Sayed, A.H.: 'Adaptive carrier tracking for Mars to Earth communications during entry, descent, and landing', *IEEE Trans. Aerosp. Electron. Syst.*, 2010, **46**, (4), pp. 1865–1879
3 Diniz, P.S.R.: 'Adaptive filtering: Algorithms and practical implementation' (Springer, Rio de Janeiro, Brazil, 2008)
4 Sayed, A.H.: 'Adaptive filters' (Wiley-IEEE Press, New Jersey, USA, 2008)
5 Regalia, P.A.: 'Adaptive IIR filtering in signal processing and control' (CRC Press, New York, USA, 1994)
6 Burt, P.M.S., and Gerken, M.: 'A polyphase IIR adaptive filter: error surface analysis and application'. IEEE Int. Conf. Acoustics, Speech, and Signal Processing, April 1997, Vol. 3, pp. 2285–2288
7 Kwan, H.K.: 'Adaptive IIR digital filters with saturation outputs for noise and echo cancellation', *Electron. Lett.*, 2002, **38**, pp. 661–663
8 So, H.C.: 'Sign-sign algorithm for unbiased IIR filtering in impulsive noise', *Electron. Lett.*, 2004, **40**, (11), pp. 704–706

9 Carini, A., Mathews, V.J., and Sicuranza, G.L.: 'Sufficient stability bounds for slowly varying direct-form recursive linear filters and their applications in adaptive IIR filters', *IEEE Trans. Signal Process.*, 1999, **47**, (9), pp. 2561–2567

10 Arenas-Garcia, J., Figueiras-Vidal, A.R., and Sayed, A.H.: 'Mean-square performance of a convex combination of two adaptive filters', *IEEE Trans. Signal Process.*, 2006, **54**, (3), pp. 1078–1090

11 Silva, M.T.M., Nascimento, V.H., and Arenas-Garcia, J.: 'A transient analysis for the convex combination of two adaptive filters with transfer of coefficients'. 2010 IEEE Int. Conf. Acoustics Speech and Signal Processing (ICASSP), Dallas, TX, USA, March 2010, pp. 3842–3845

12 Luk, F.T., and Qiao, S.: 'A fast eigenvalue algorithm for Hankel matrices', *Linear Algebr. Appl.*, 2000, **316**, (1), pp. 171–182, Special issue for the 60th birthday of R.J. Plemmons

13 Wang, L.-x., Xiang, W., and Zhang, J.-y.: 'The Monte Carlo framework for adaptive IIR filter and arma parameters estimation'. IET Conf. Wireless, Mobile and Sensor Networks, Shanghai, China, December 2007, pp. 725–728